Finite Automata

Part Two

Outline for Today

- ***Recap from Last Time***
  - Where are we, again?
- ***Designing a DFA***
  - How to think about finite memory.
- ***Regular Languages***
  - A fundamental class of languages.
- ***NFAs***
  - Automata with Magic Superpowers.
- ***Designing NFAs***
  - Harnessing an awesome power.

Recap from Last Time

Formal Language Theory

- An ***alphabet*** is a set, usually denoted Σ, consisting of elements called ***characters***.
  - $a \in \Sigma$ means "*a* is a single character."
- A ***string over Σ*** is a finite sequence of zero or more characters taken from Σ.
- The ***empty string*** has no characters and is denoted ε.
- A ***language over Σ*** is a set of strings over Σ.
- The language **Σ*** is the set of all strings over Σ.
  - $w \in \Sigma^*$ means "*w* is a string of characters from Σ."

The Language of an Automaton

- If A is an automaton that processes strings over Σ, the ***language of A***, denoted $\mathcal{L}(A)$, is the set of all strings A accepts.
- Formally:

$$\mathcal{L}(A) = \{ w \in \Sigma^* \mid A \text{ accepts } w \}$$

DFAs

- A ***DFA*** is a
  - ***D***eterministic
  - ***F***inite
  - ***A***utomaton
- DFAs are the simplest type of automaton that we will see in this course.

DFAs

- A DFA is defined relative to some alphabet Σ.
- For each state in the DFA, there must be ***exactly one*** transition defined for each symbol in Σ.
  - This is the "deterministic" part of DFA.
- There is a unique start state.
- There are zero or more accepting states.

New Stuff!

Designing DFAs

- At each point in its execution, the DFA can only remember what state it is in.
- ***DFA Design Tip:*** Build each state to correspond to some piece of information you need to remember.
  - Each state acts as a “memento” of what you're supposed to do next.
  - Only finitely many different states means only finitely many different things the machine can remember.

Recognizing Languages with DFAs

$L = \{ w \in \{$**a**, **b**$\}^* \mid$ the number of **b**'s in w is congruent to two modulo three $\}$

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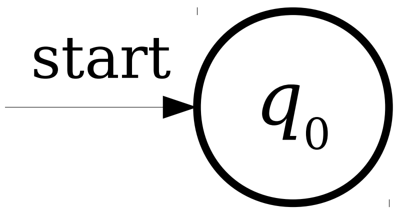

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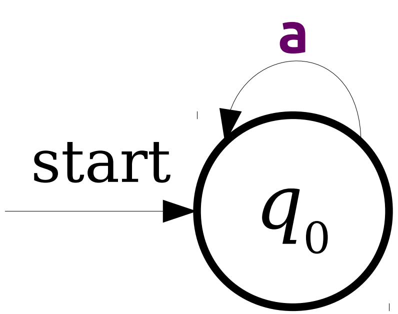

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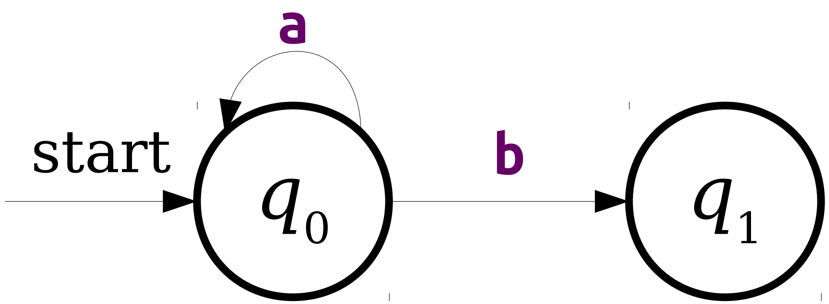

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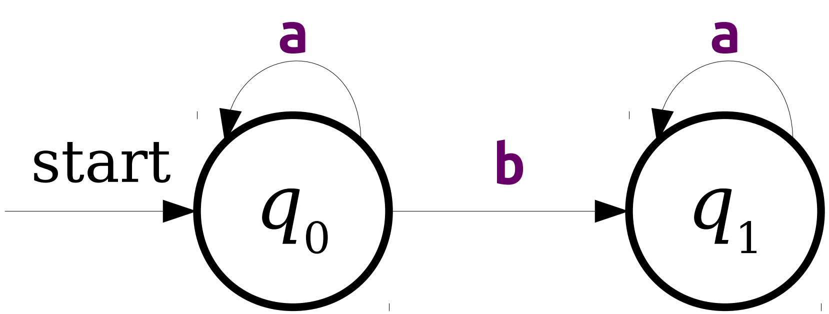

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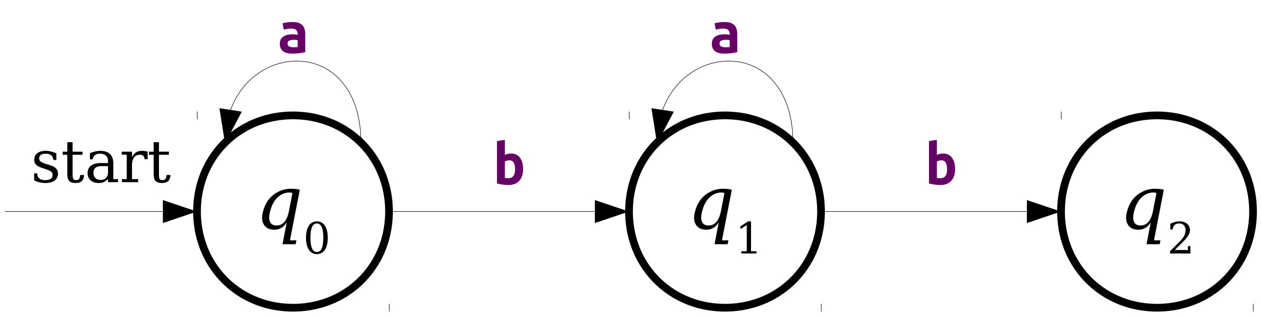

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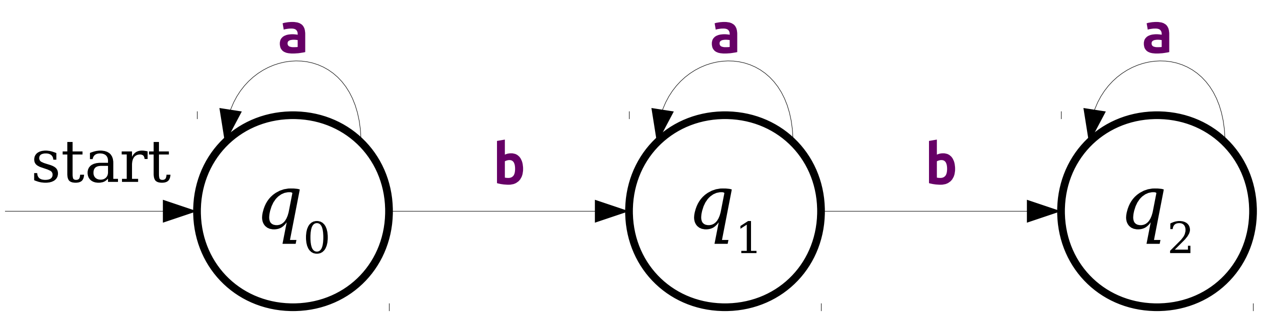

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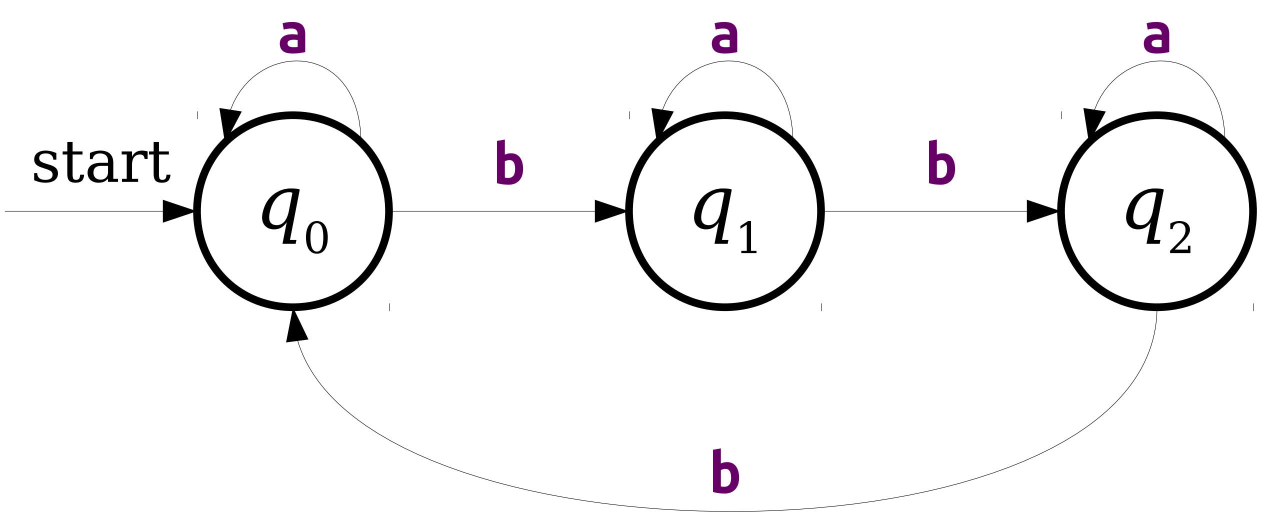

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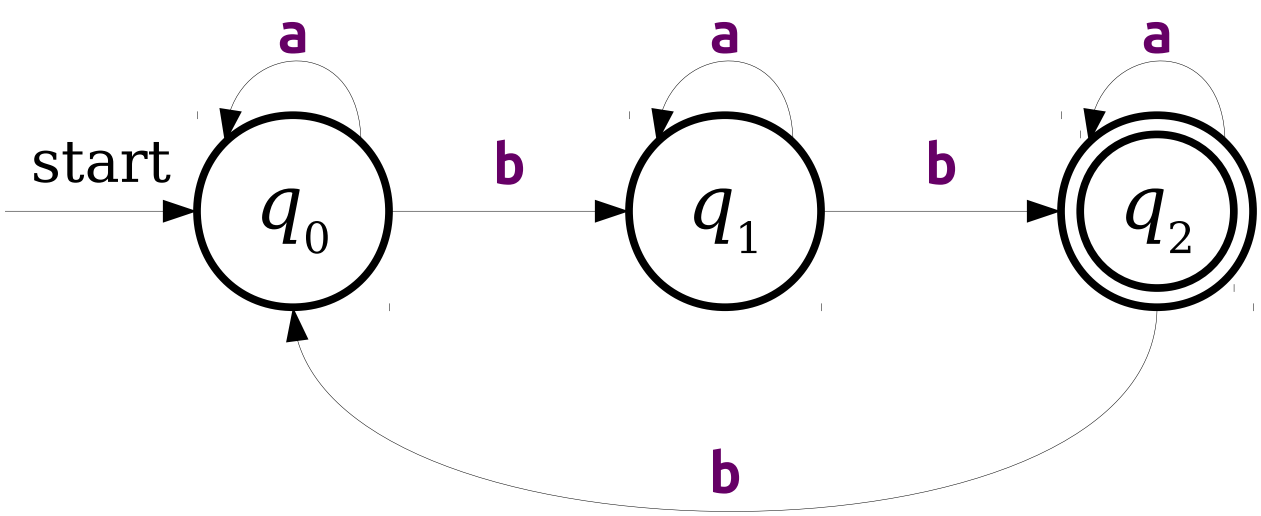

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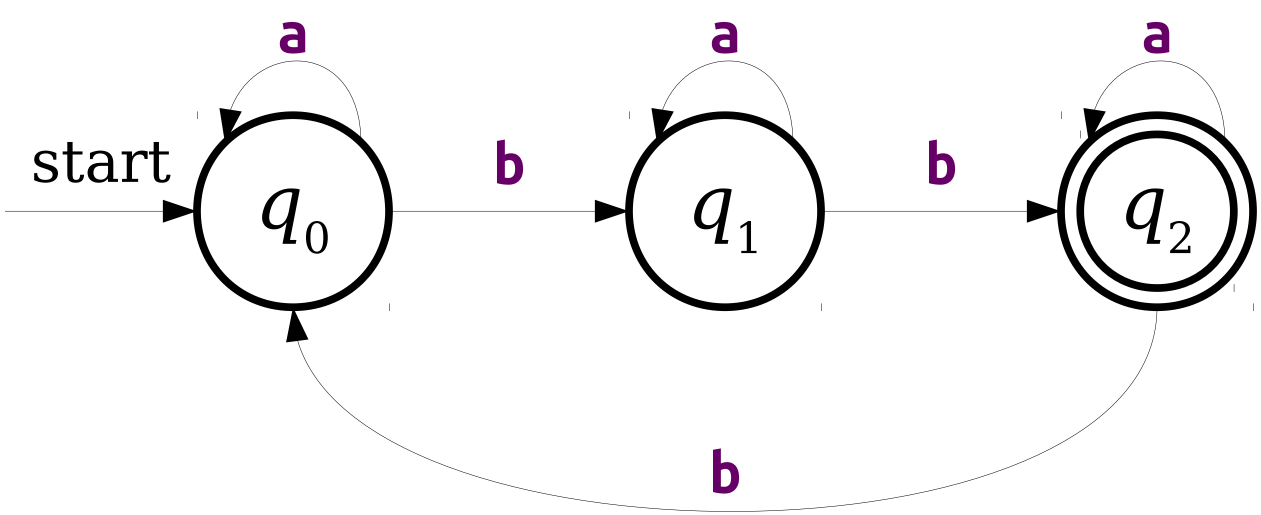

Each state remembers the remainder of the number of **b**s seen so far modulo three.

Recognizing Languages with DFAs

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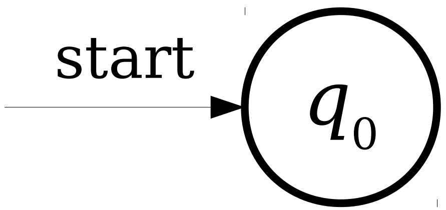

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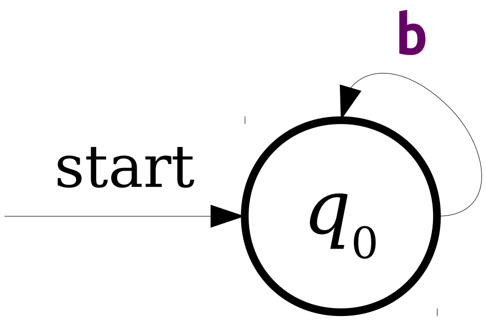

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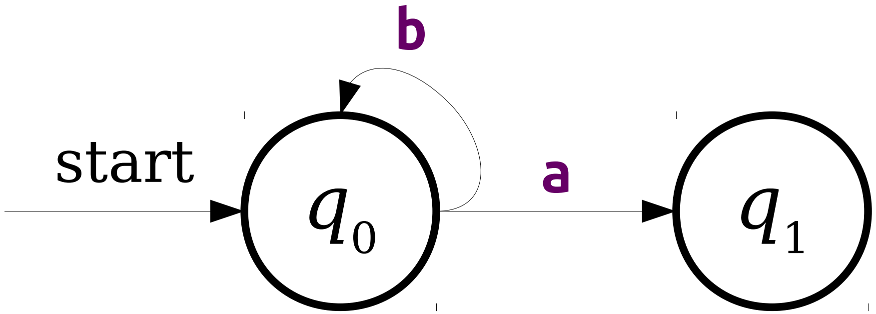

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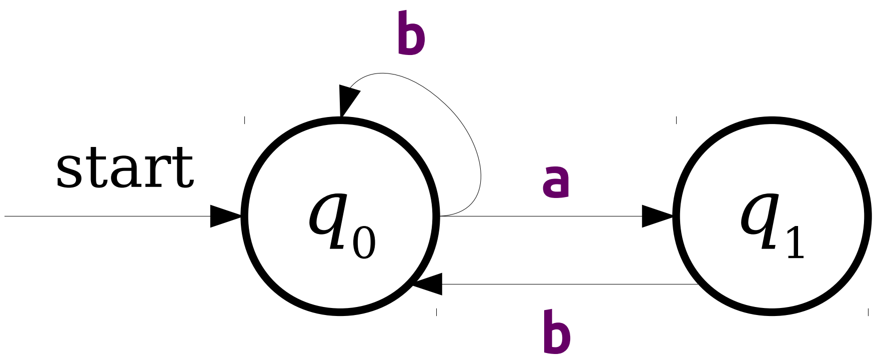

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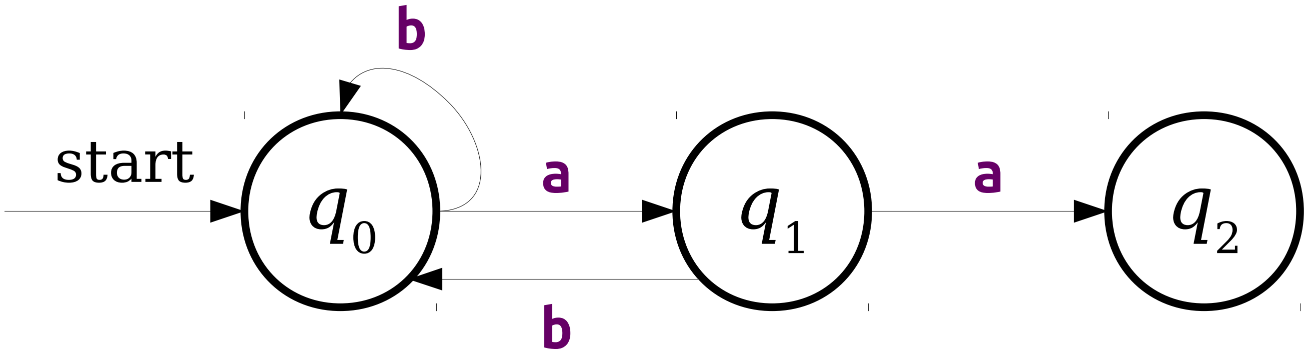

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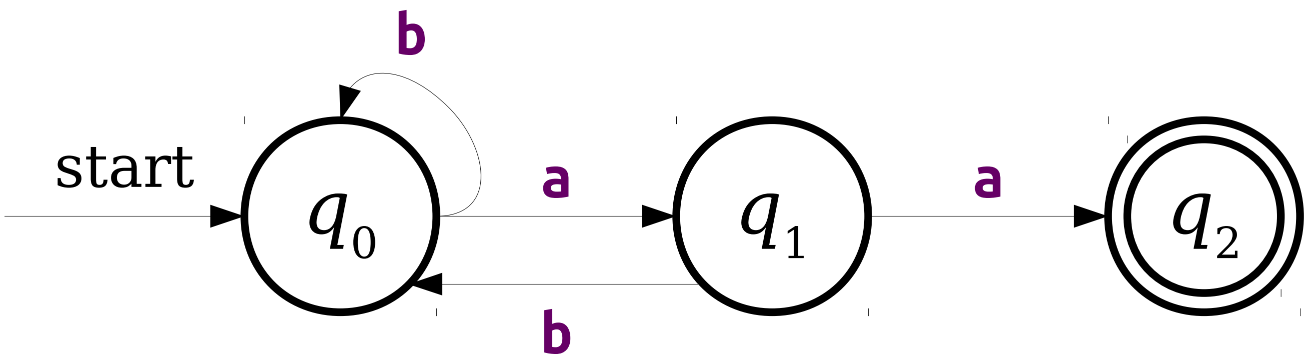

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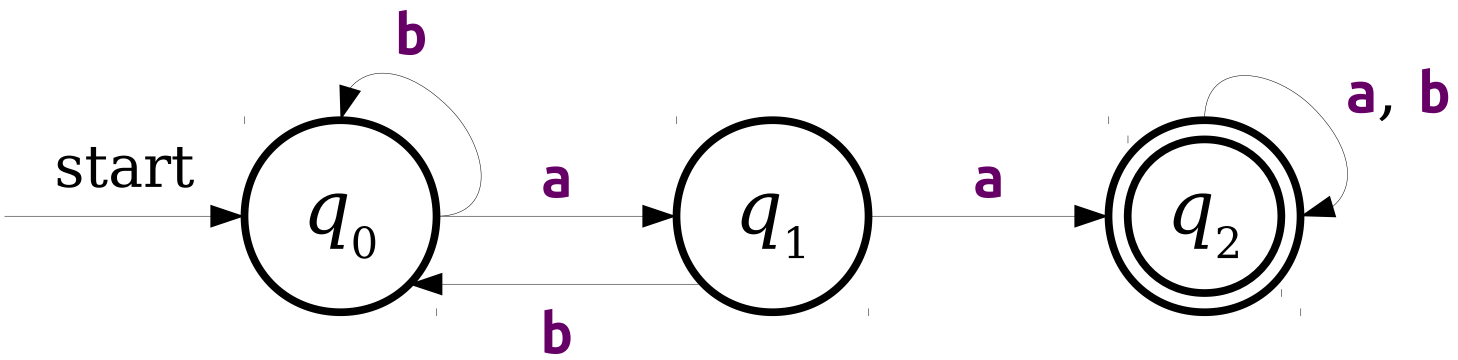

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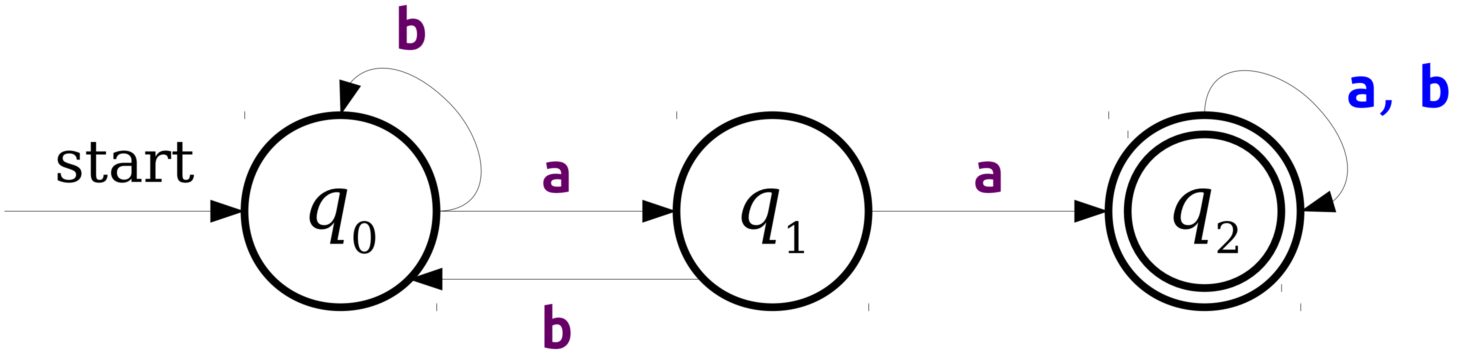

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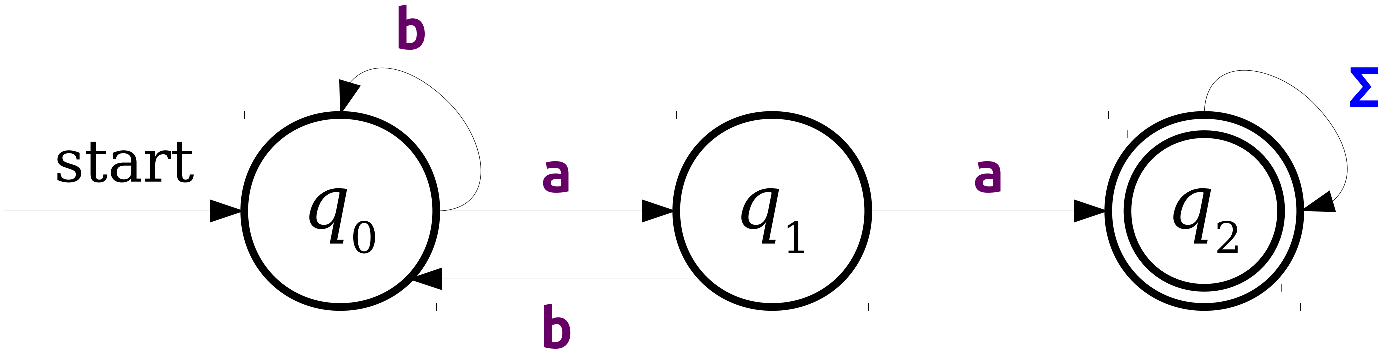

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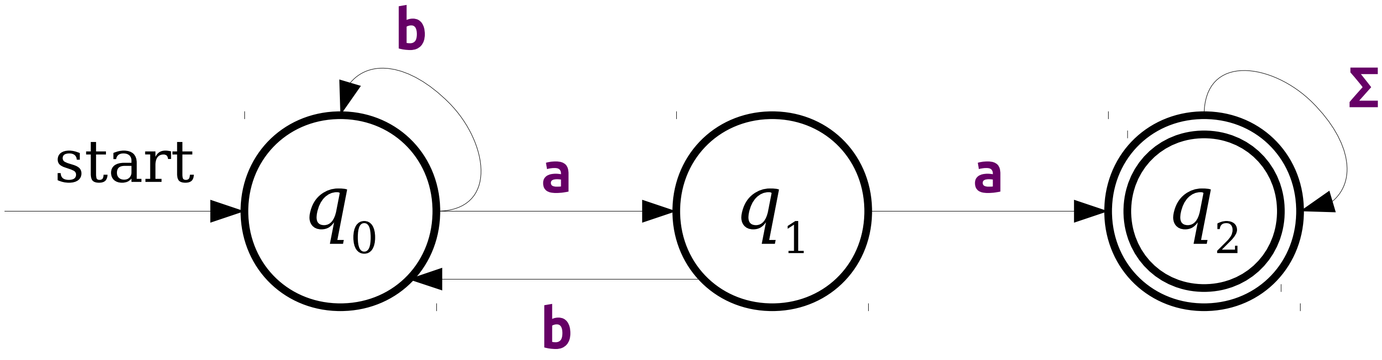

The Regular Languages

A language L is called a ***regular language*** if there exists a DFA D such that $\mathcal{L}(D) = L$.

If L is a language and $\mathcal{L}(D) = L$, we say that D ***recognizes*** the language L.

The Complement of a Language

- Given a language $L \subseteq \Sigma^*$, the ***complement*** of that language (denoted $\overline{L}$) is the language of all strings in Σ^* that aren't in L.
- Formally:

$$\overline{L} = \Sigma^* - L$$

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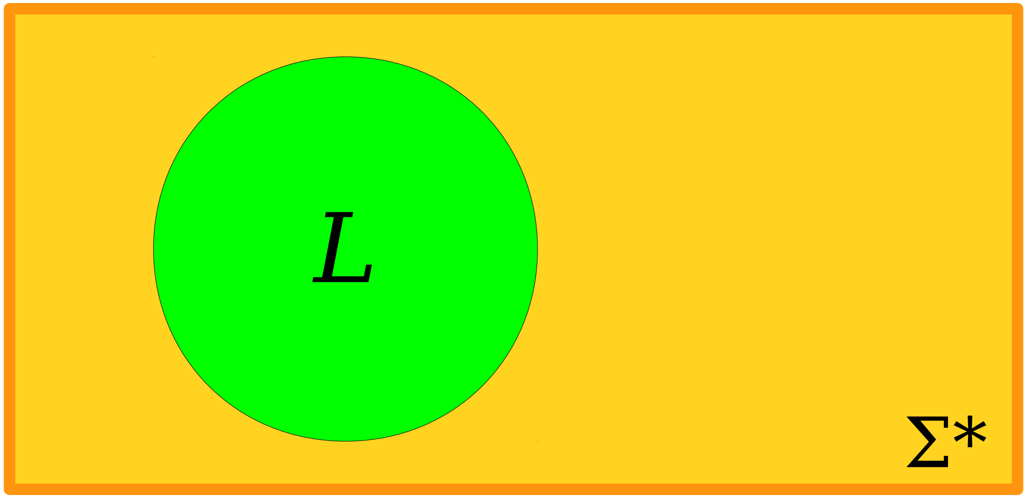

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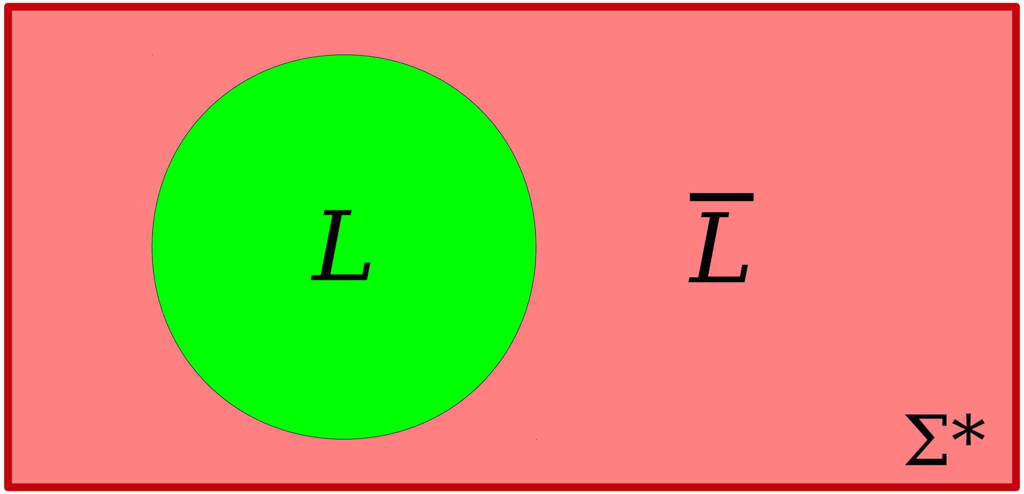

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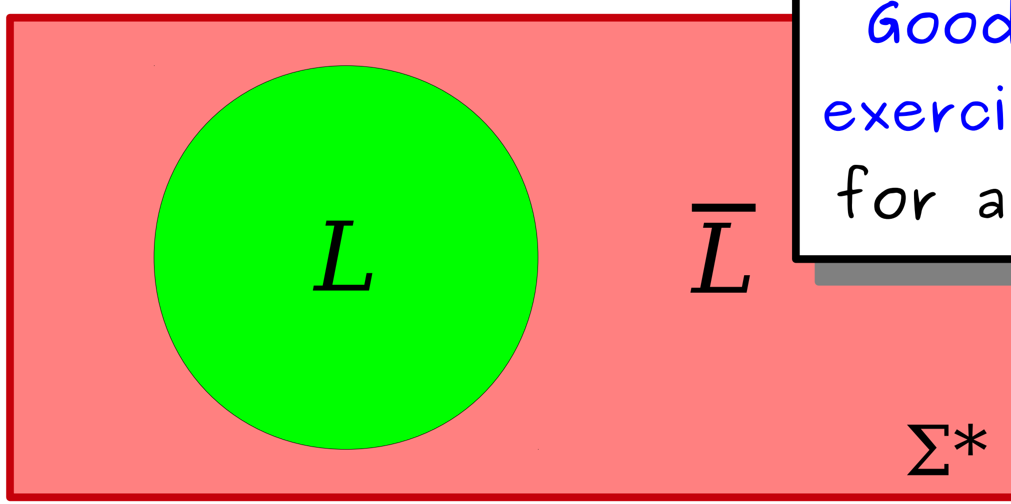
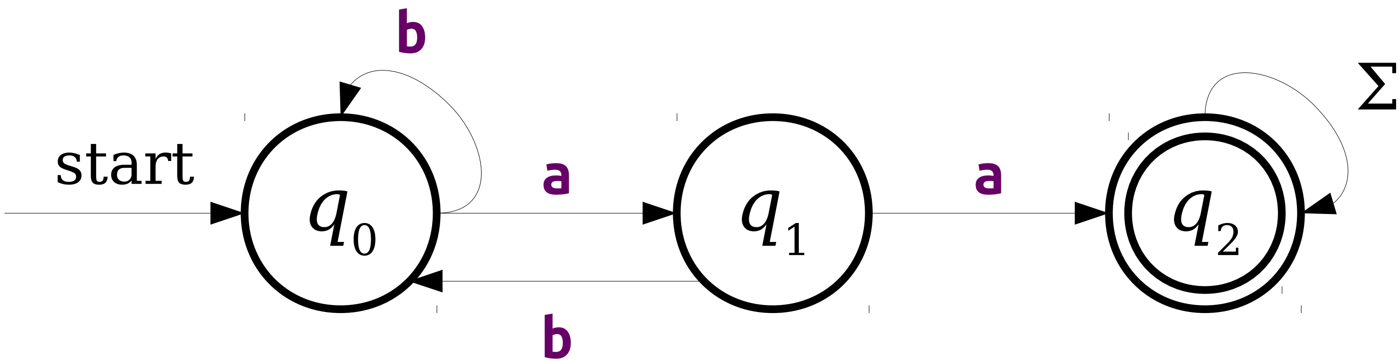


Good proofwriting exercise: prove $\overline{\overline{L}} = L$ for any language L.

Complementing Regular Languages

$L = \{ w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w \text{ contains } \mathtt{aa} \text{ as a substring } \}$



$\overline{L} = \{ w \in \{\mathtt{a}, \mathtt{b}\}^* \mid w$ ***does not*** $\text{contain } \mathtt{aa} \text{ as a substring } \}$

How do we turn the DFA above into a DFA for $\overline{L}$?

Answer at
https://pollev.com/cs103

Complementing Regular Languages

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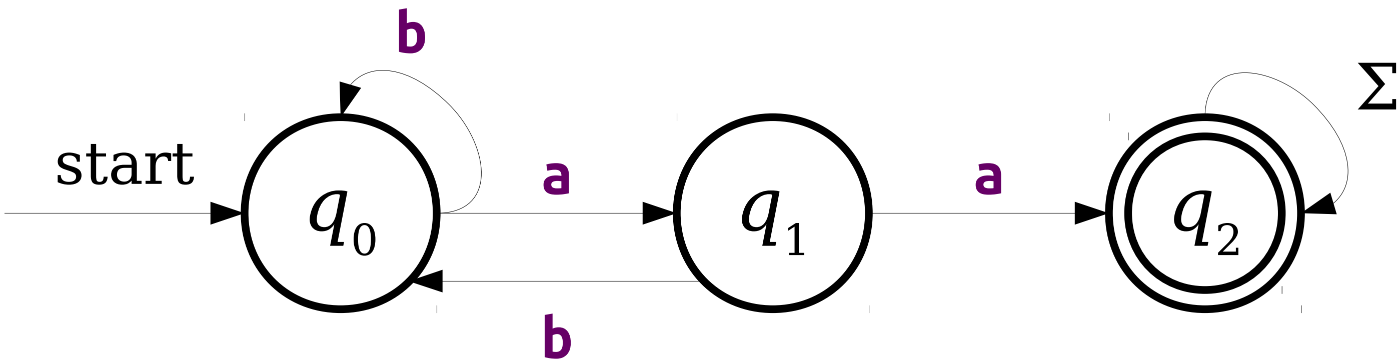

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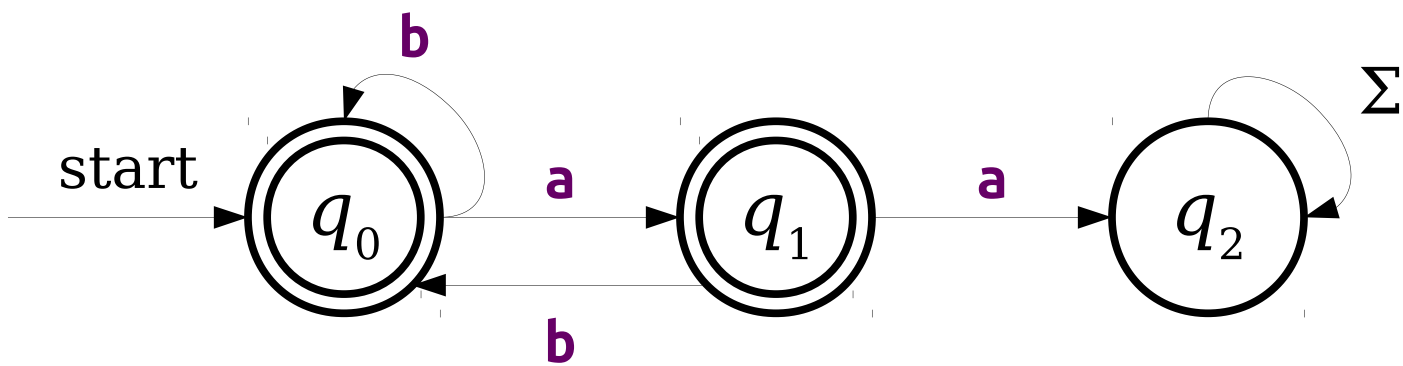
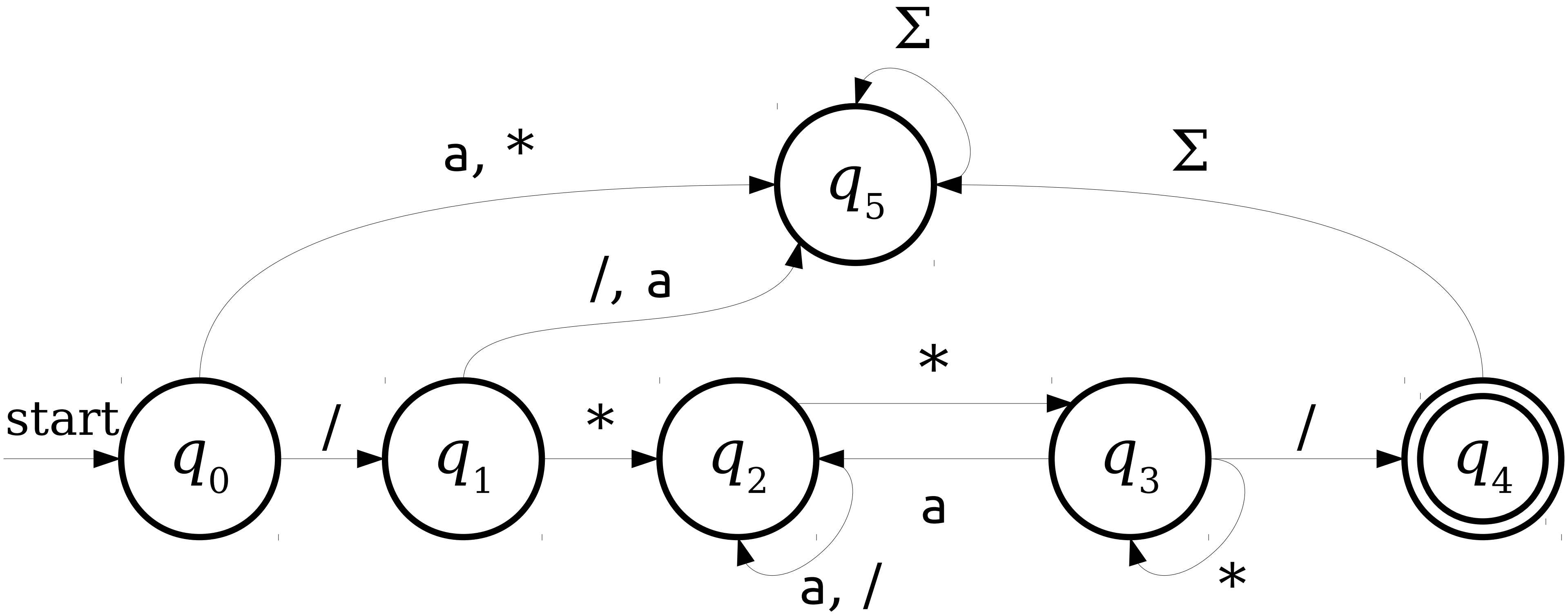


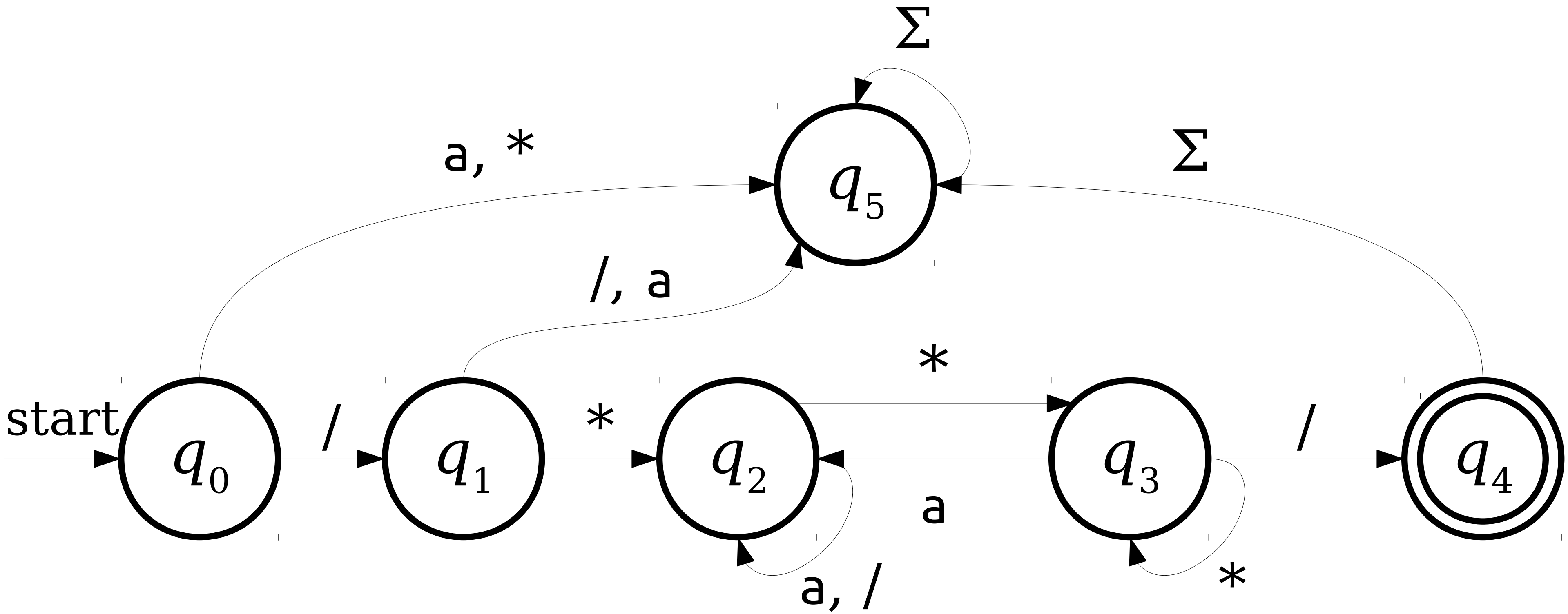
Complementing Regular Languages

$L = \{ w \in \{$**a**, *****, **/**$\}^* \mid w$ represents a C-style comment $\}$



Complementing Regular Languages

$\overline{L}$ = { w ∈ {**a**, *****, **/**}* | w ***doesn't*** represent a C-style comment }



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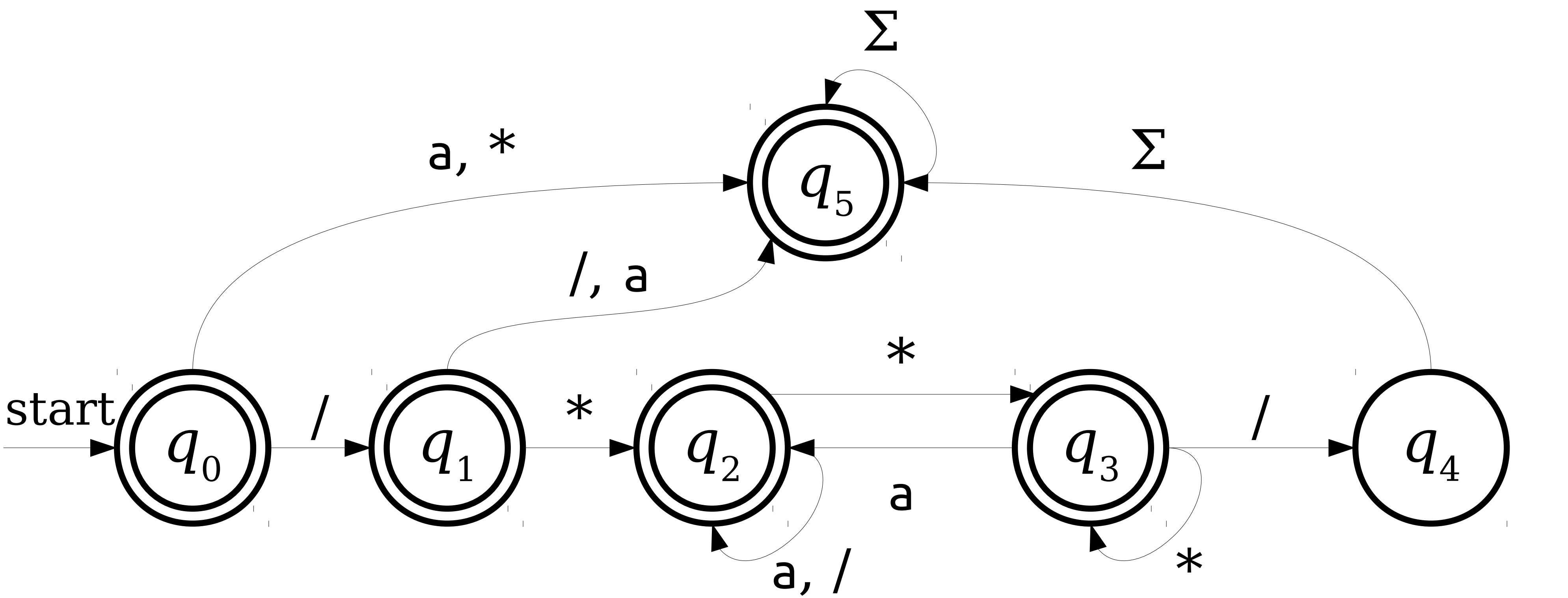
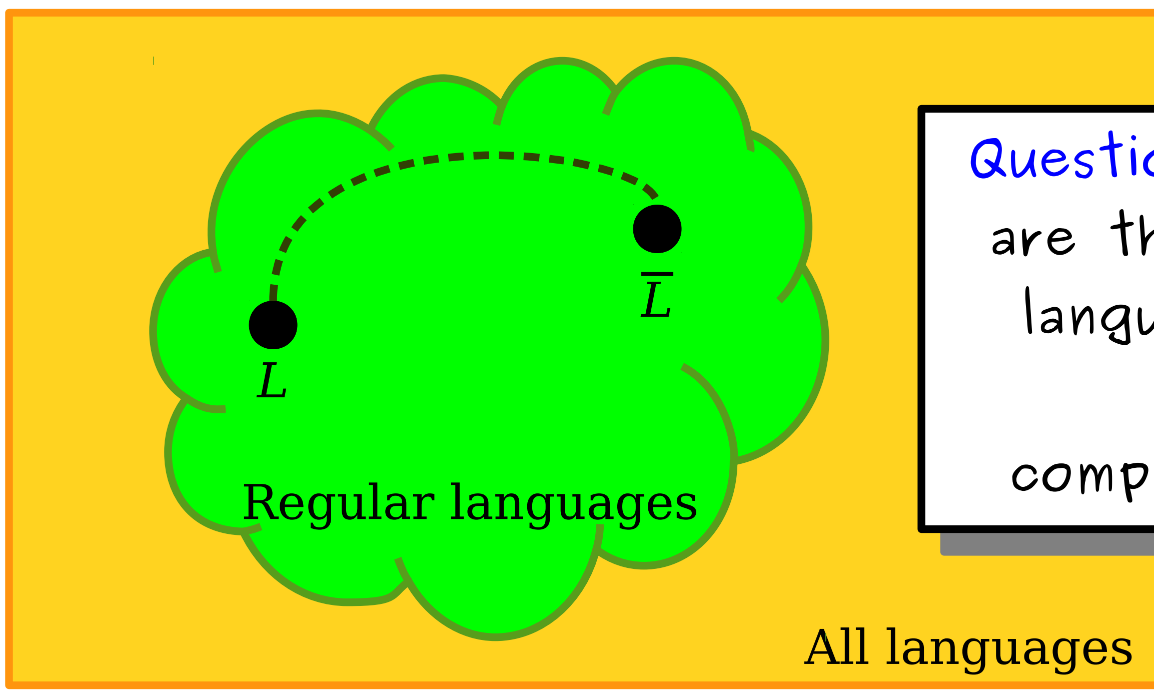


Closure Properties

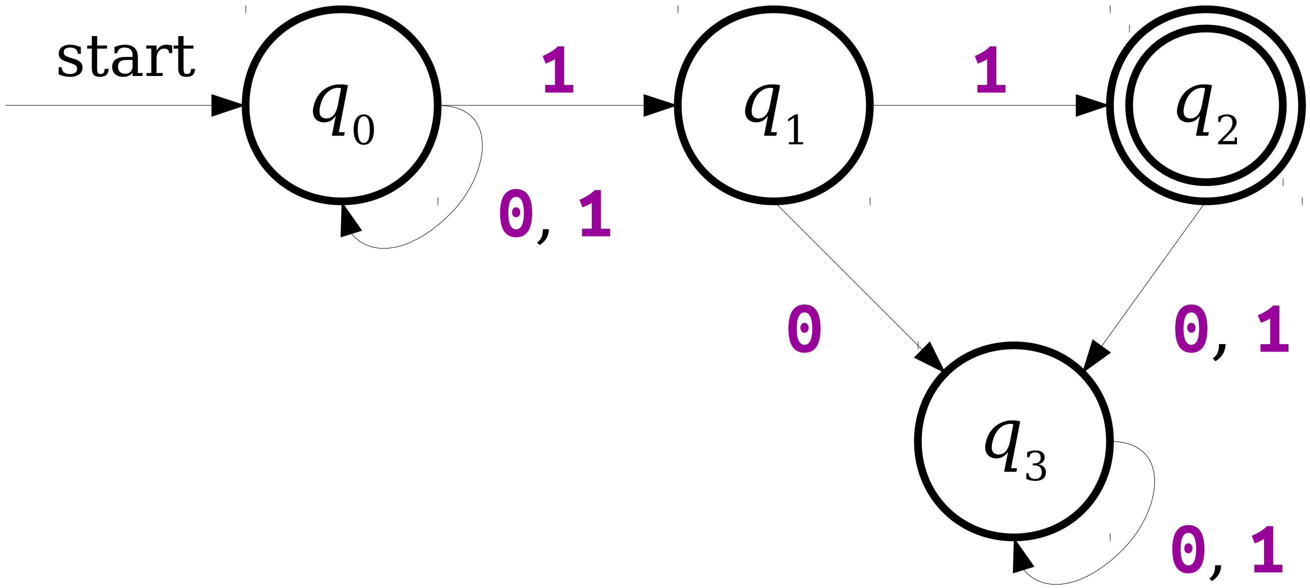
- ***Theorem:*** If L is a regular language, then $\overline{L}$ is also a regular language.
- As a result, we say that the regular languages are ***closed under complementation***.



Question to ponder: are the *nonregular* languages closed under complementation?

NFAs

The Motivation



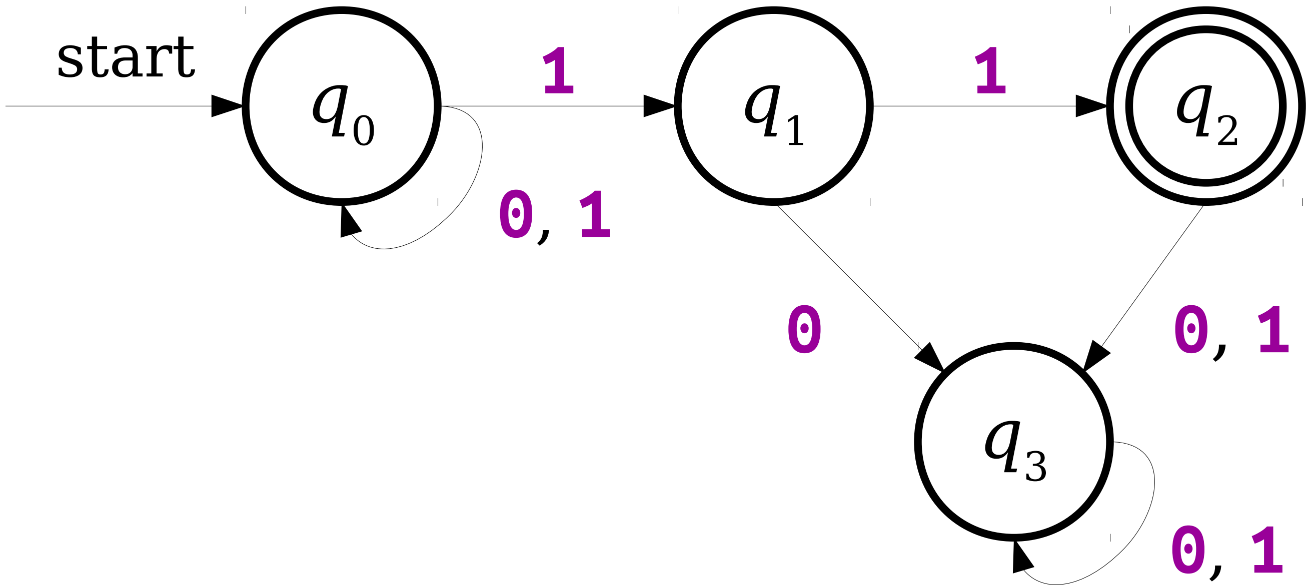
NFAs

- An ***NFA*** is a
  - ***N***ondeterministic
  - ***F***inite
  - ***A***utomaton
- Structurally similar to a DFA, but represents a fundamental shift in how we'll think about computation.

(Non)determinism

- A model of computation is ***deterministic*** if at every point in the computation, there is exactly one choice that can make.
  - The machine accepts if that series of choices leads to an accepting state.
- A model of computation is ***nondeterministic*** if the computing machine has a finite number of choices available to make at each point, possibly including zero.
- The machine accepts if ***any*** series of choices leads to an accepting state.
  - (This sort of nondeterminism is technically called ***existential nondeterminism***, the most philosophical-sounding term we'll introduce all quarter.)

A Simple NFA



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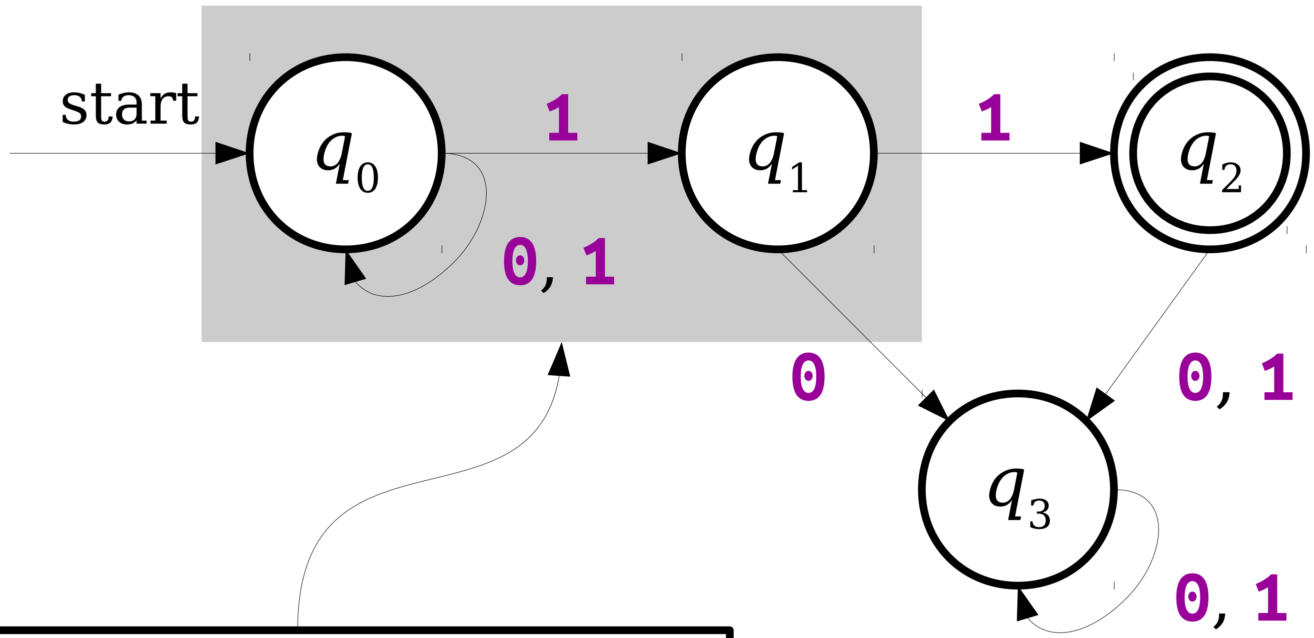

q_0 has two transitions defined on 1!

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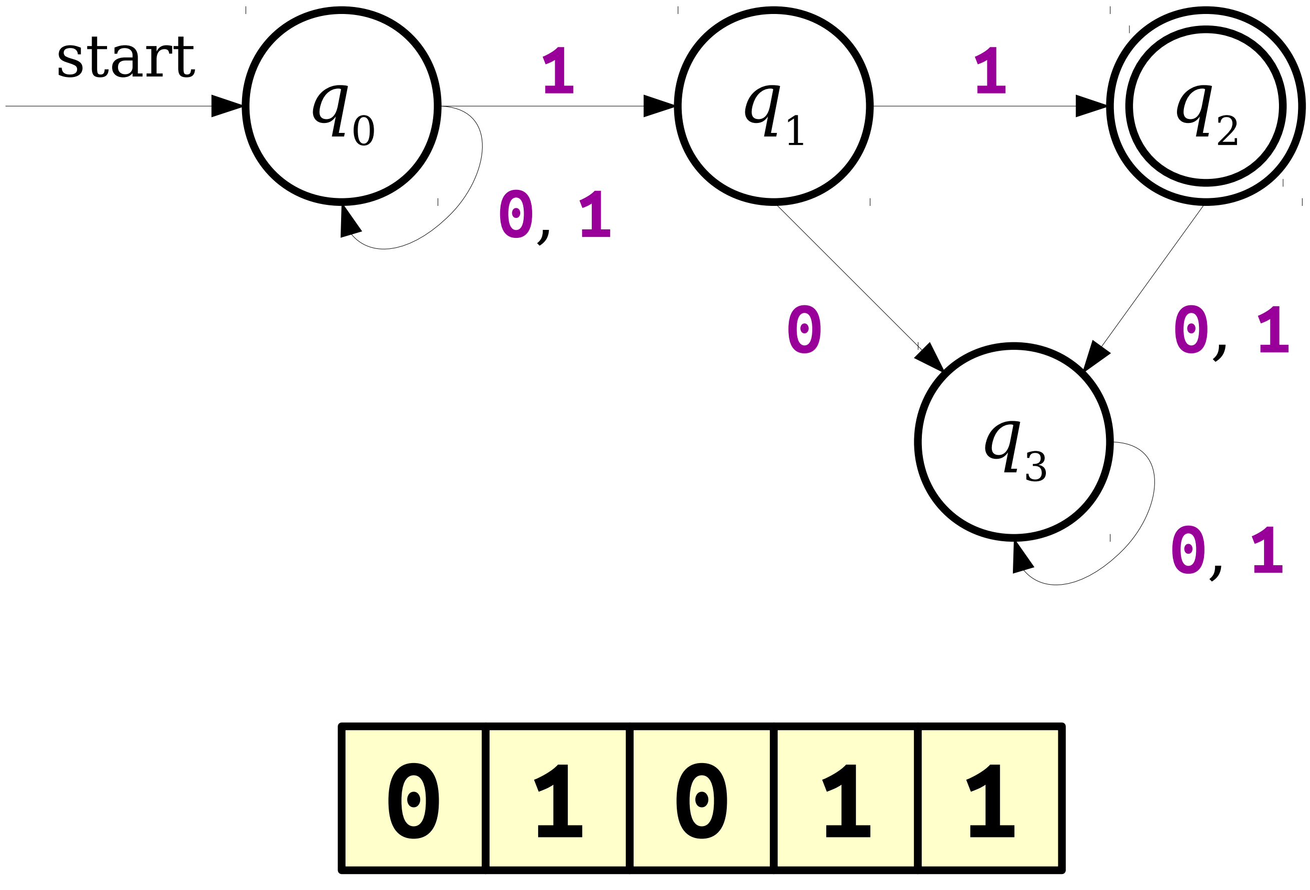

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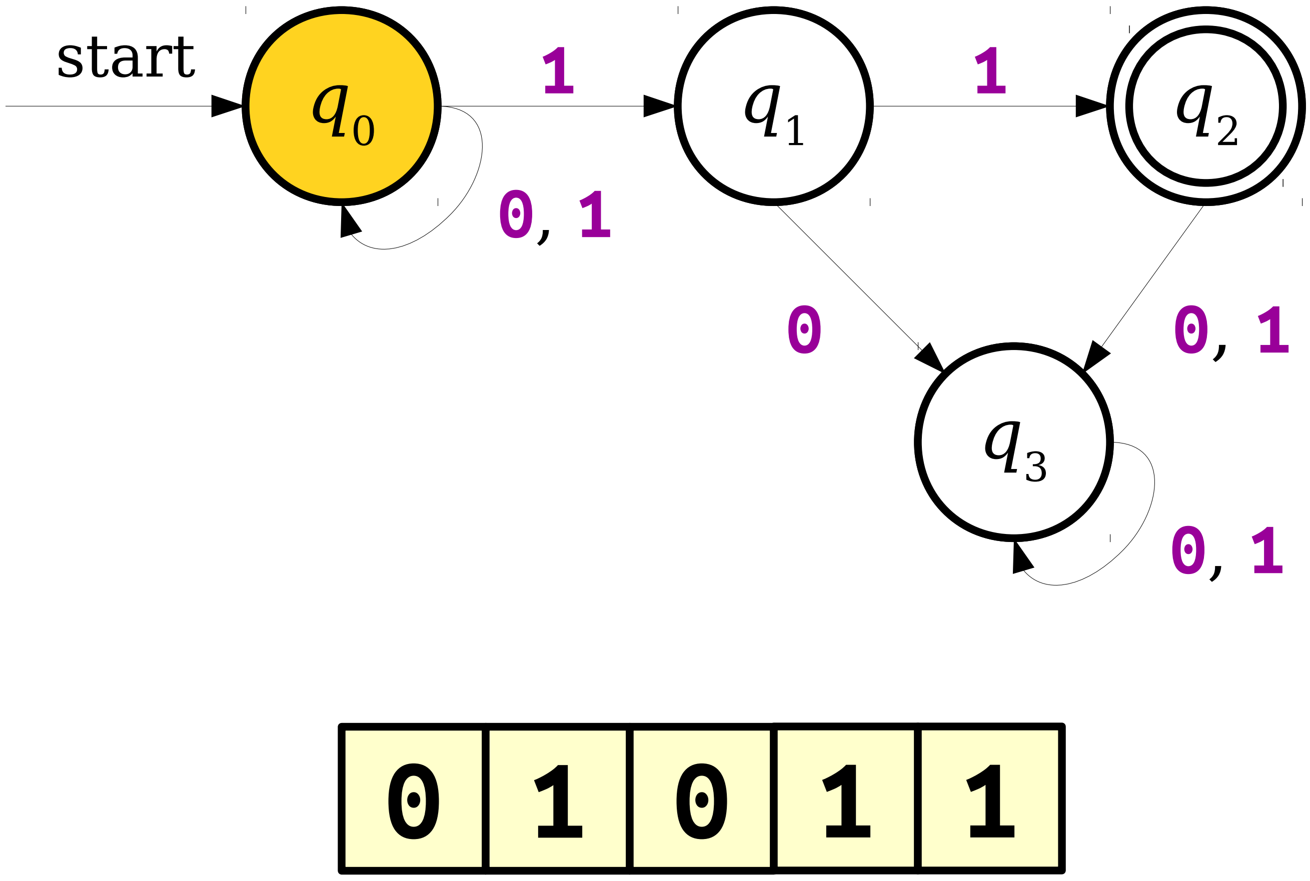

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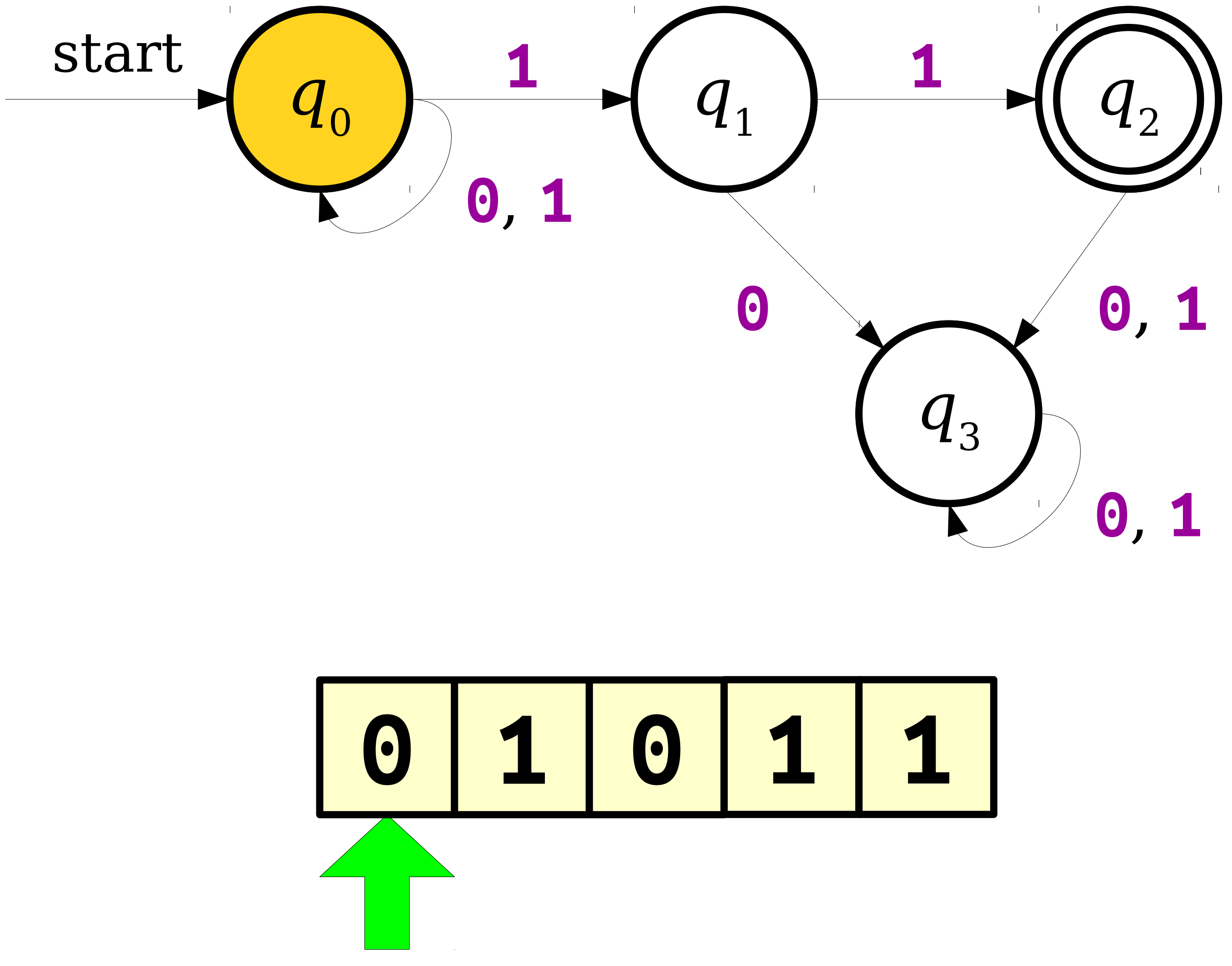

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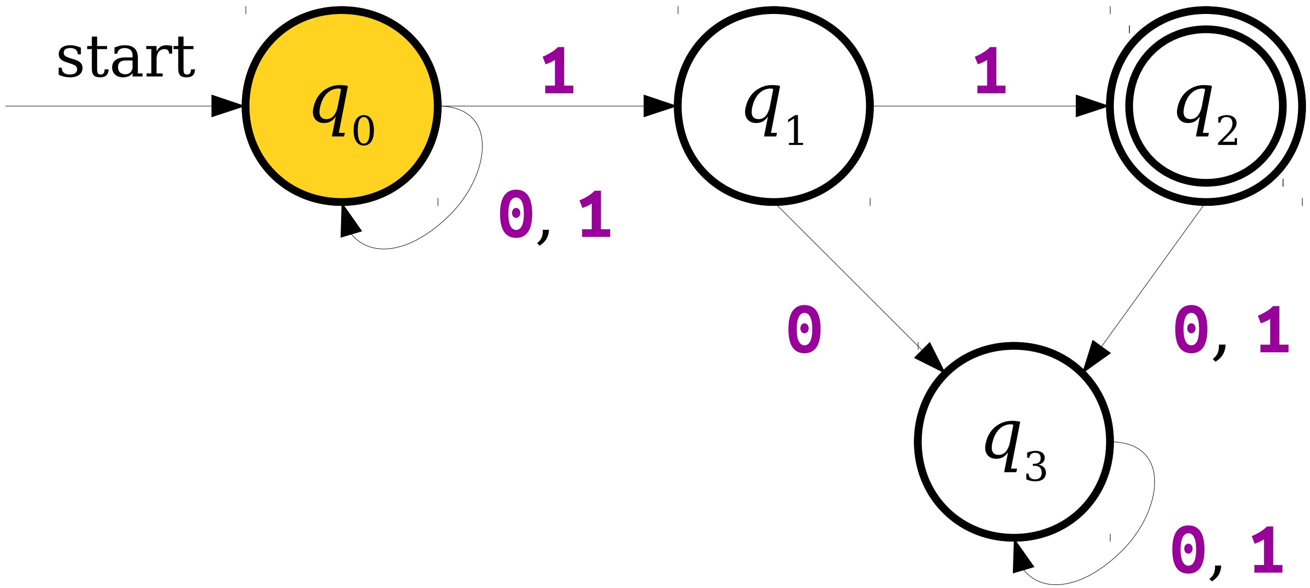

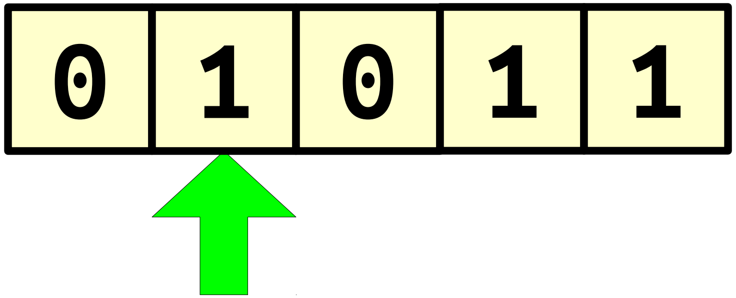

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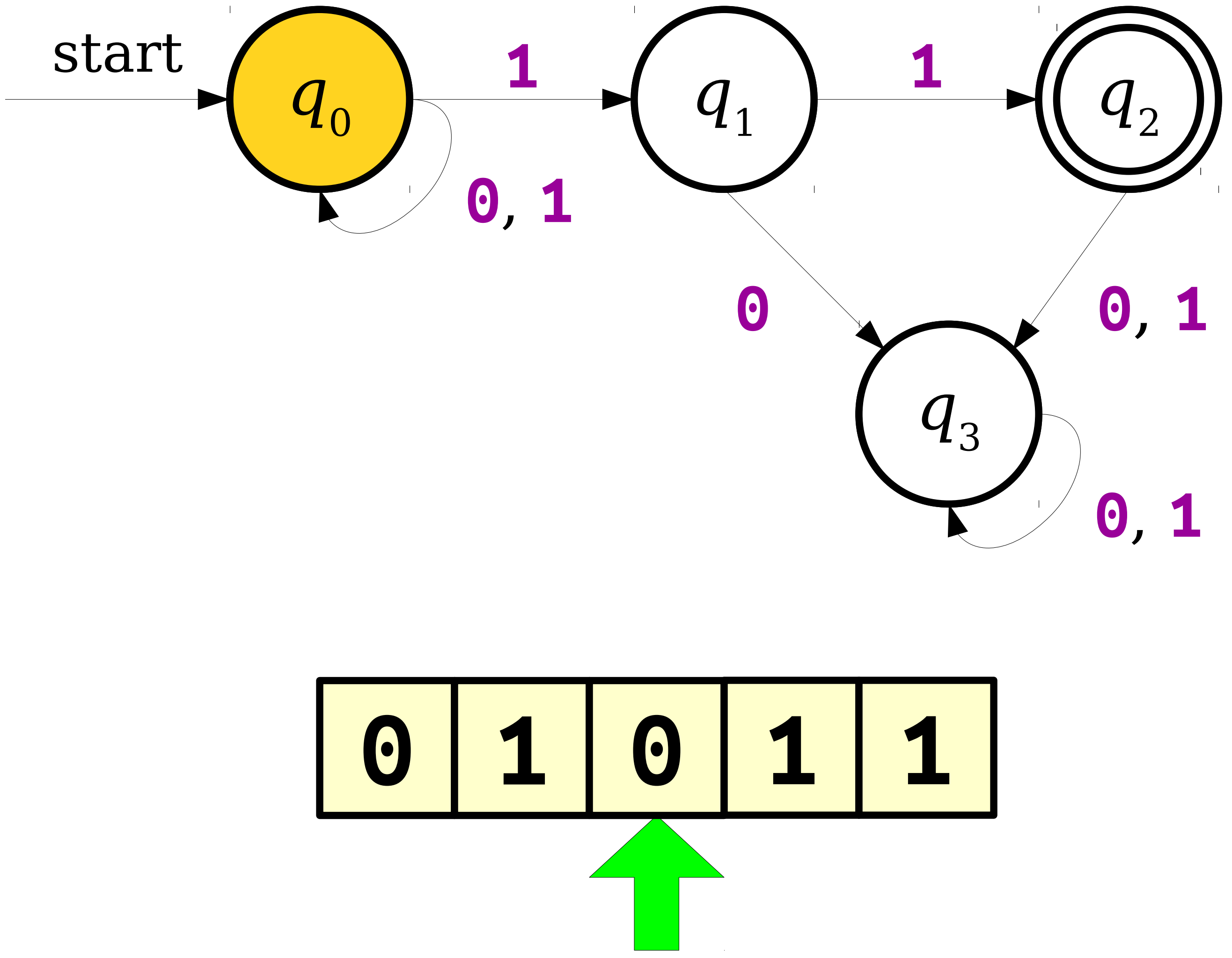

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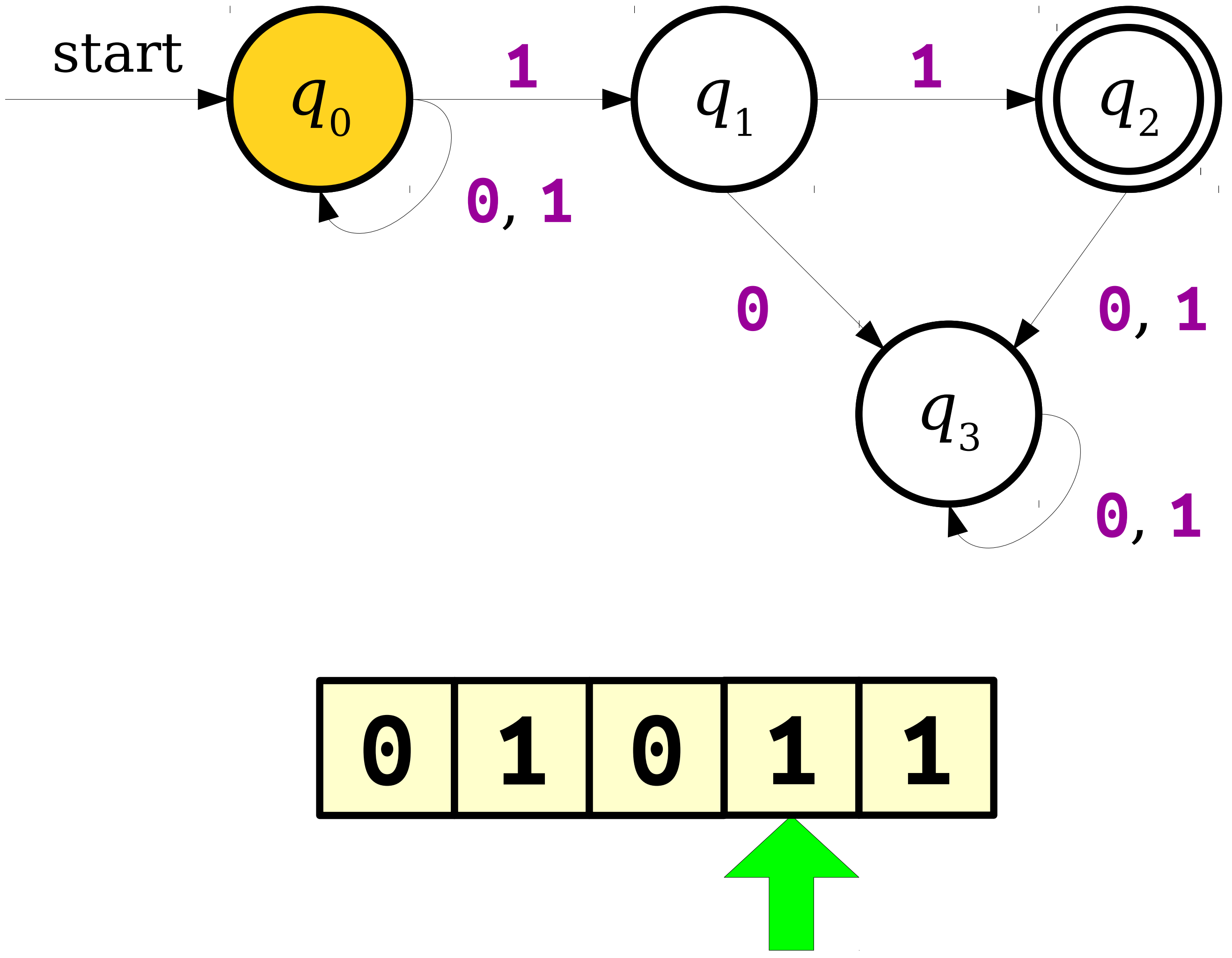

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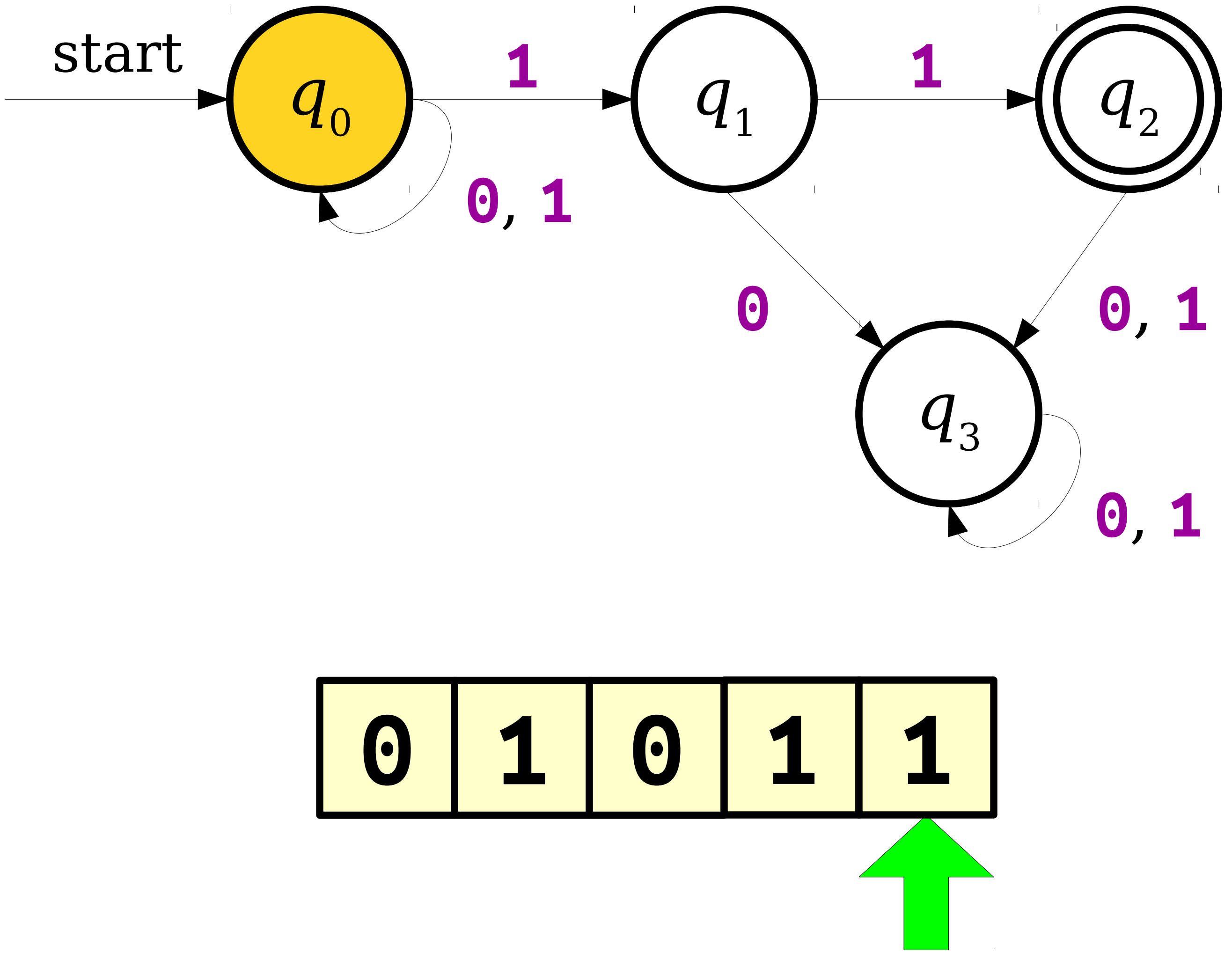

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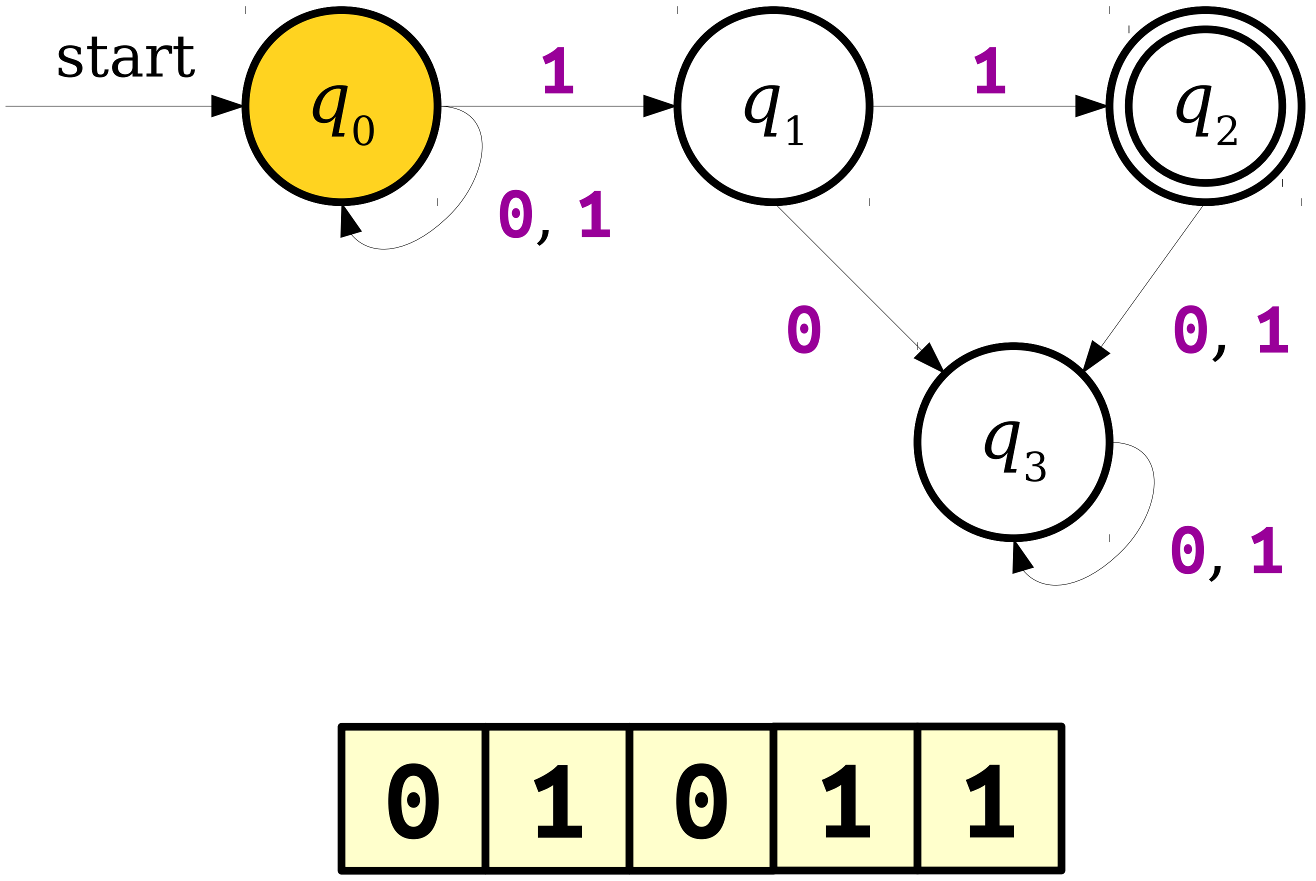

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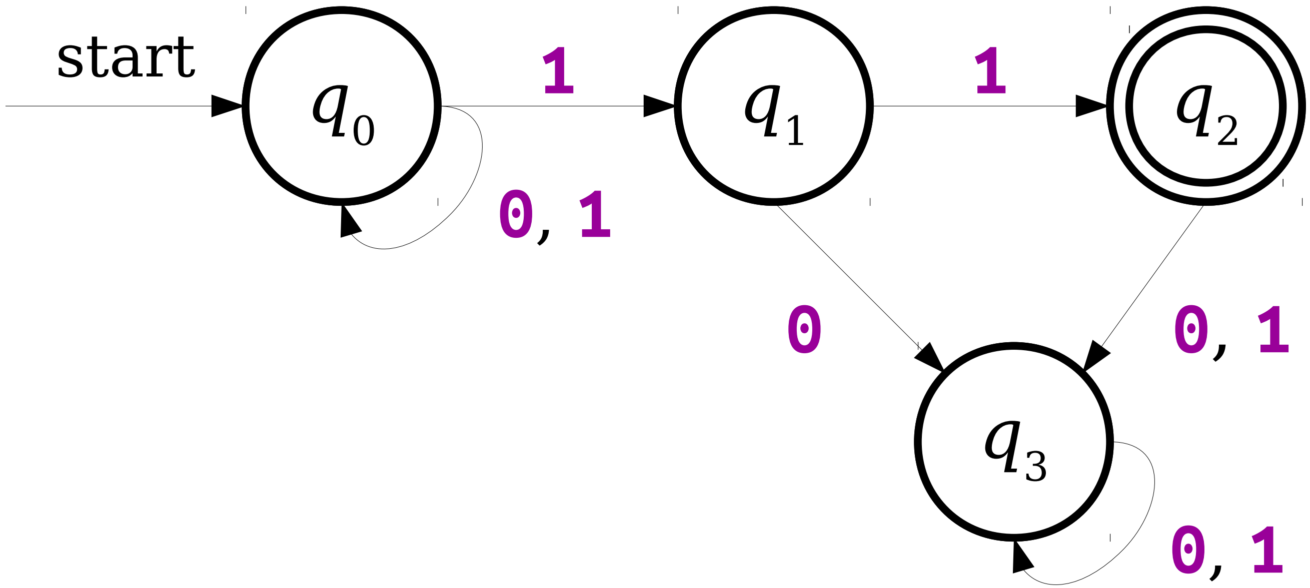

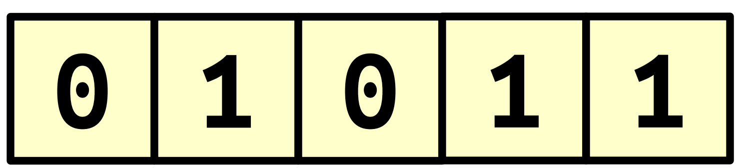

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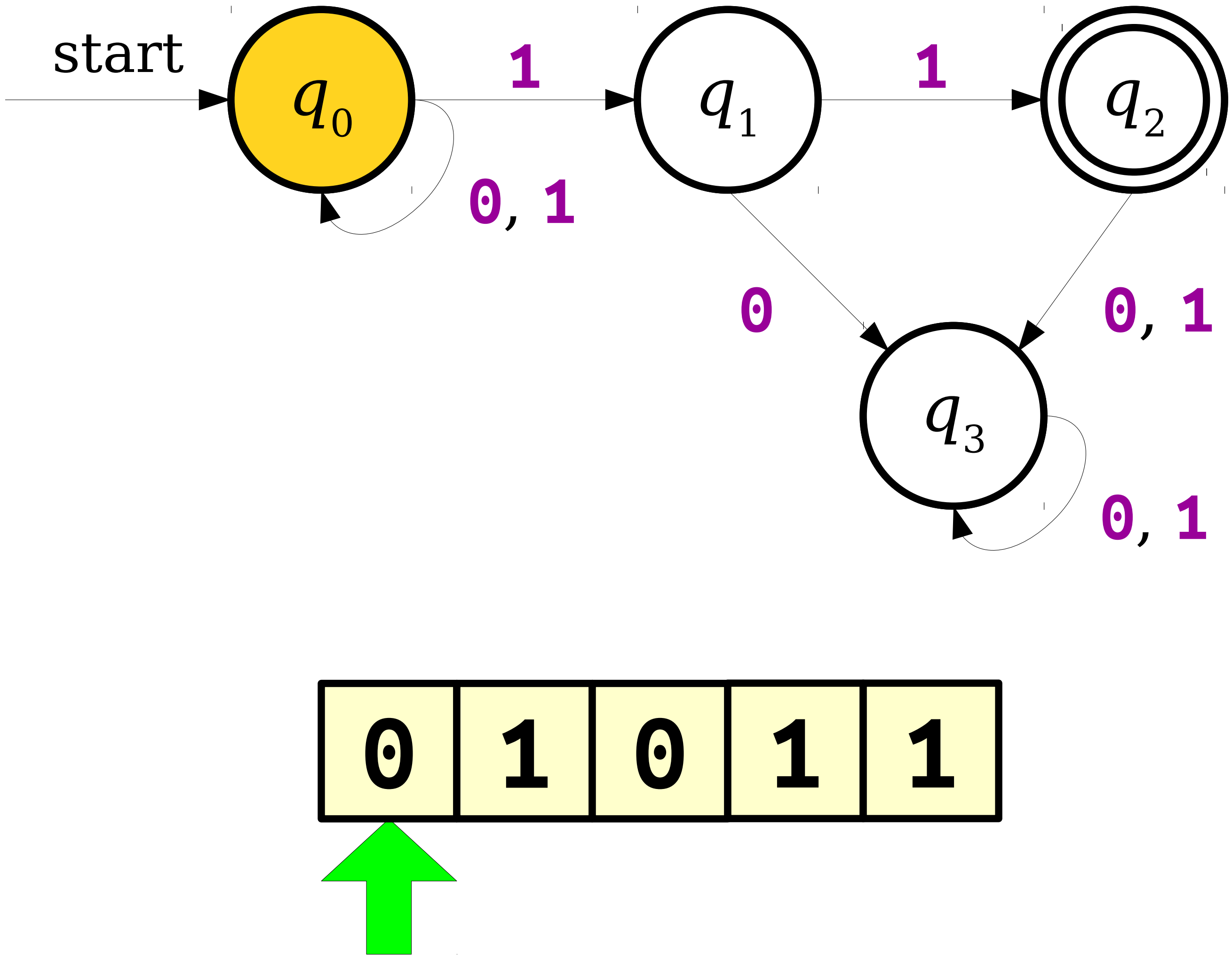

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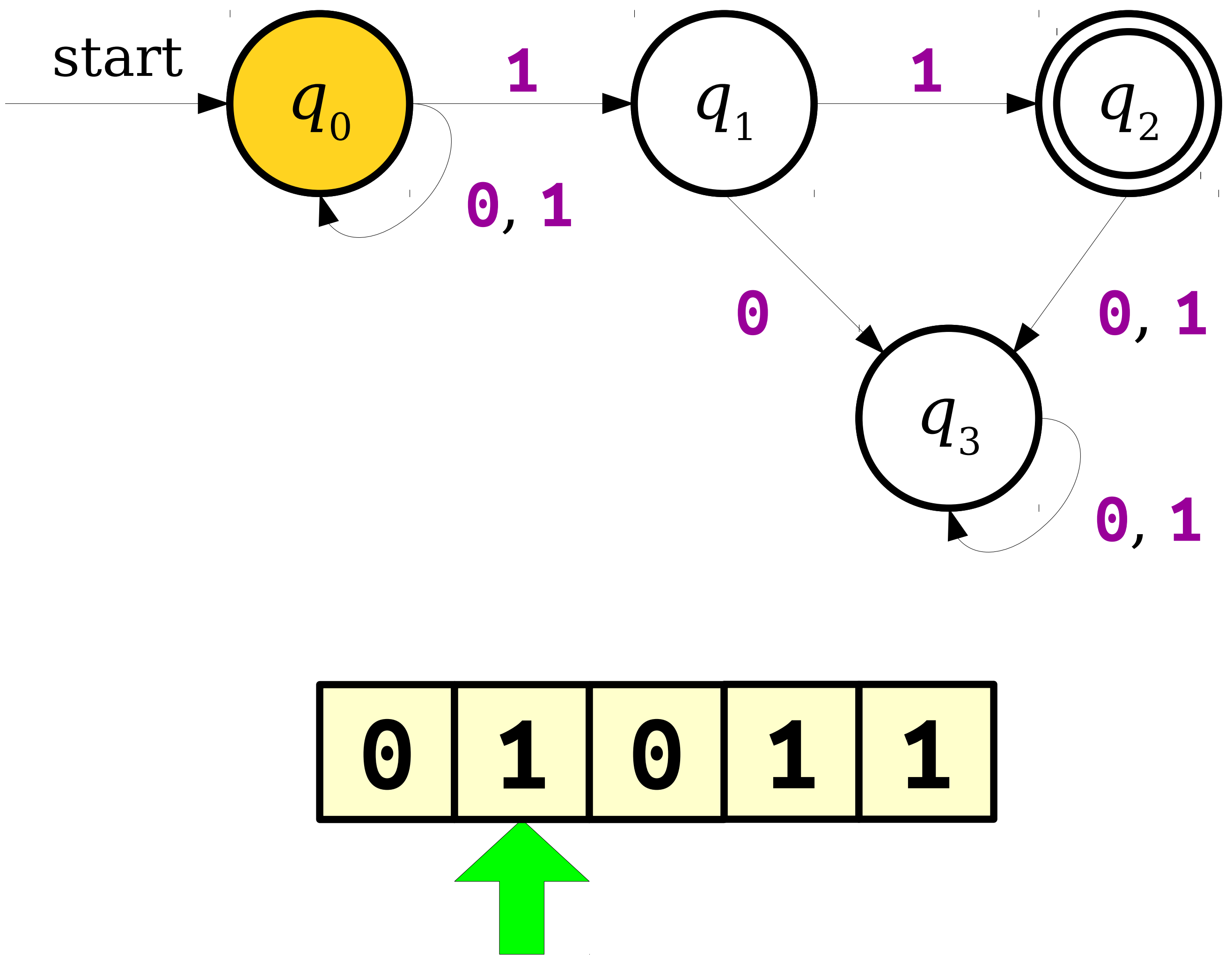

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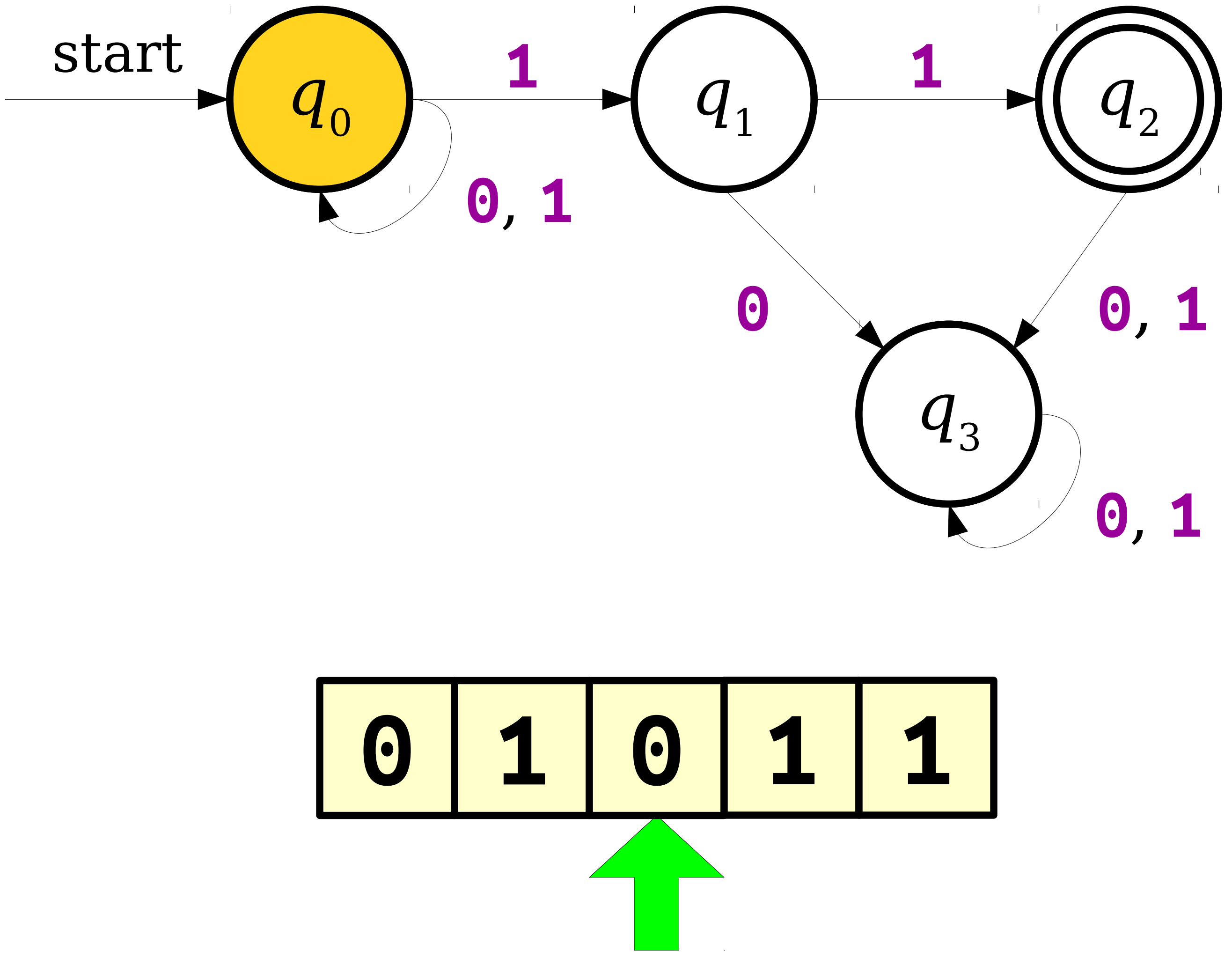

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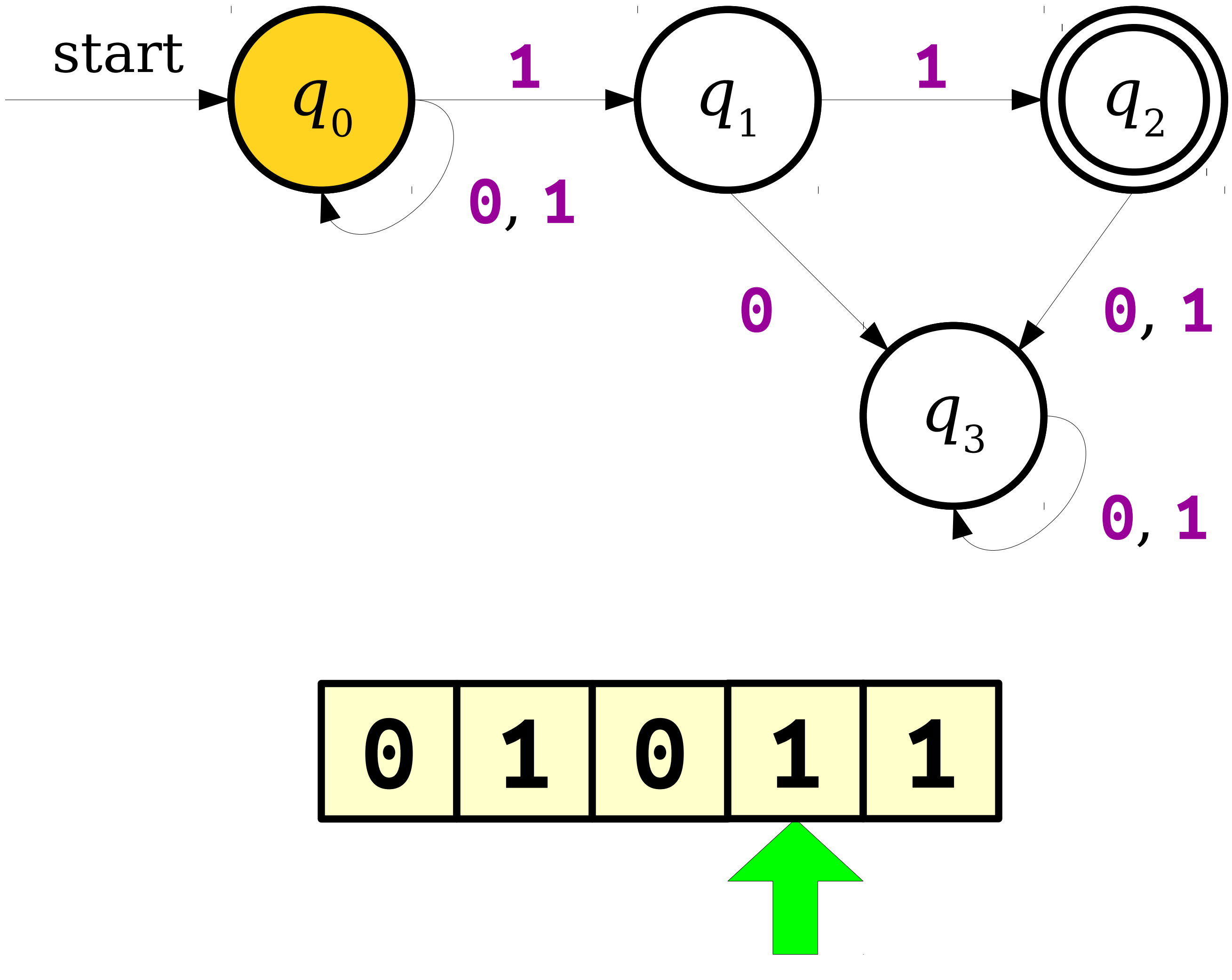

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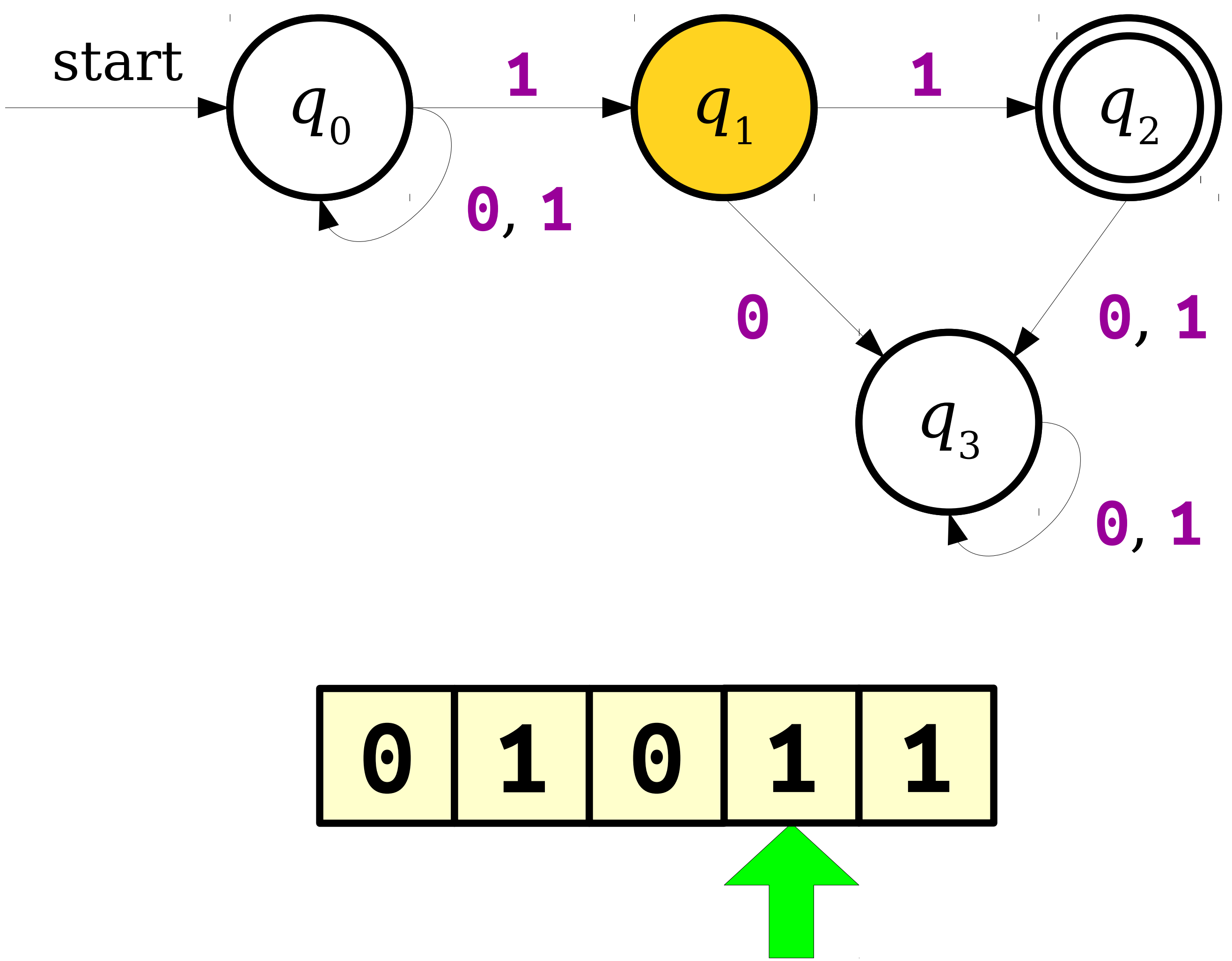

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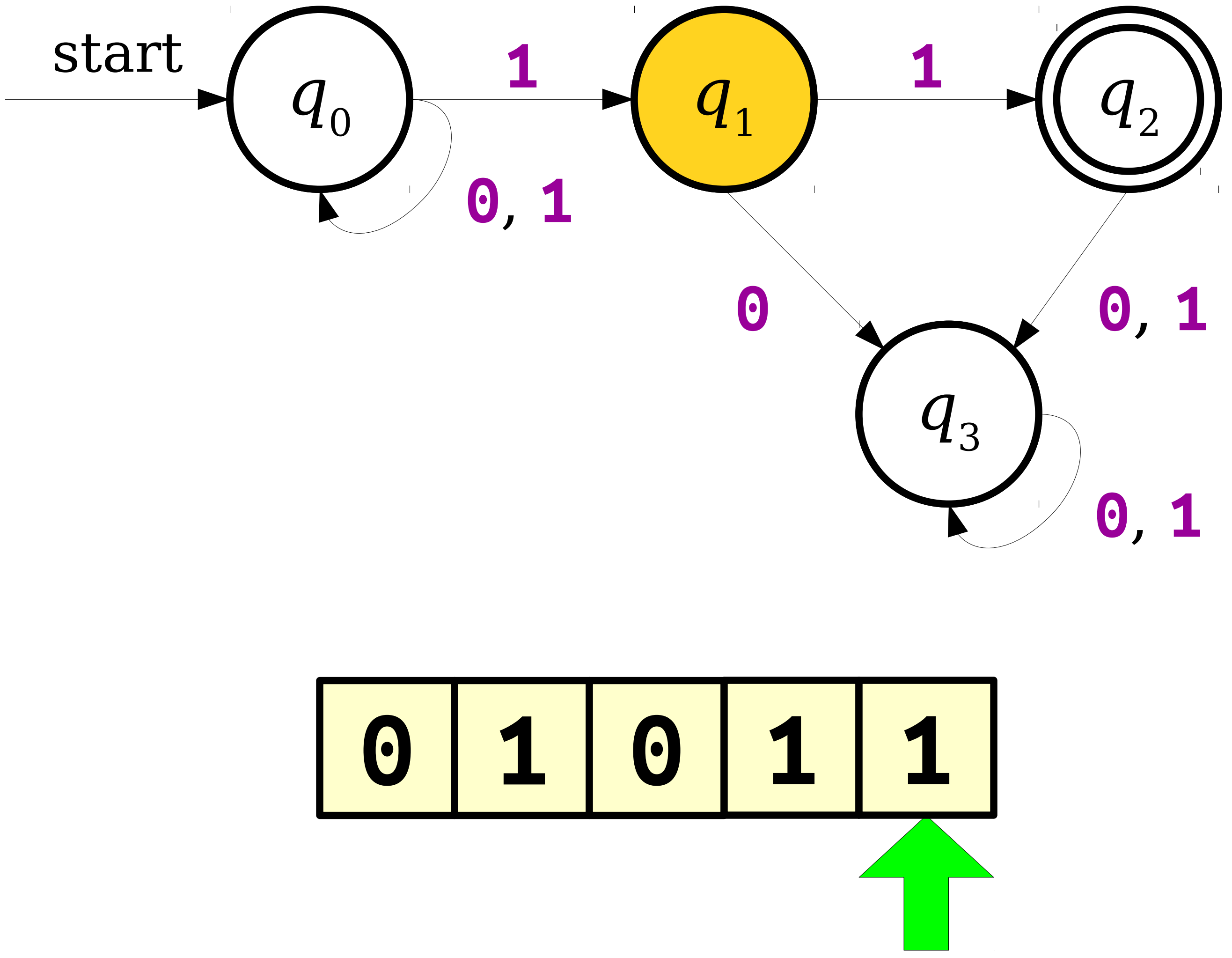

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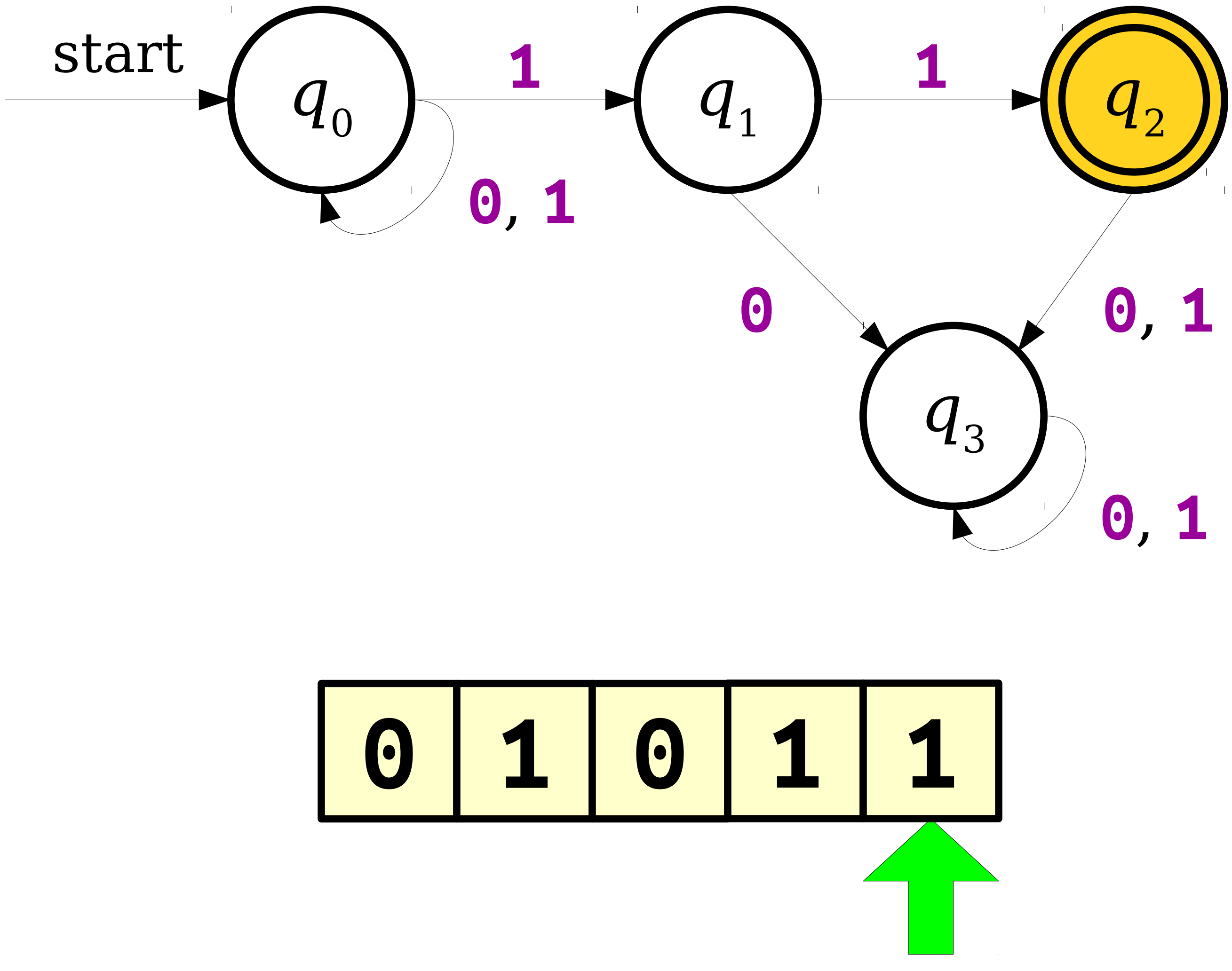

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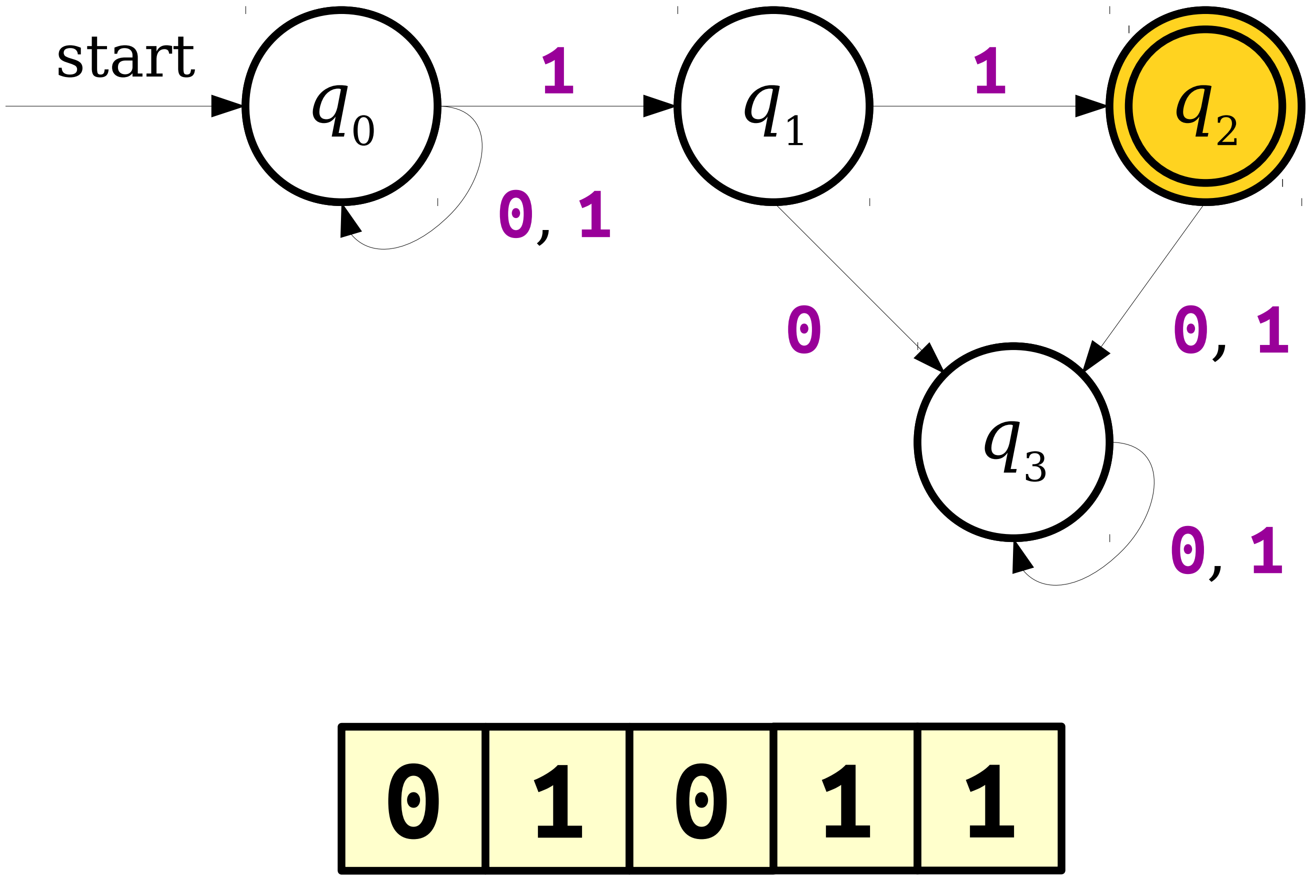

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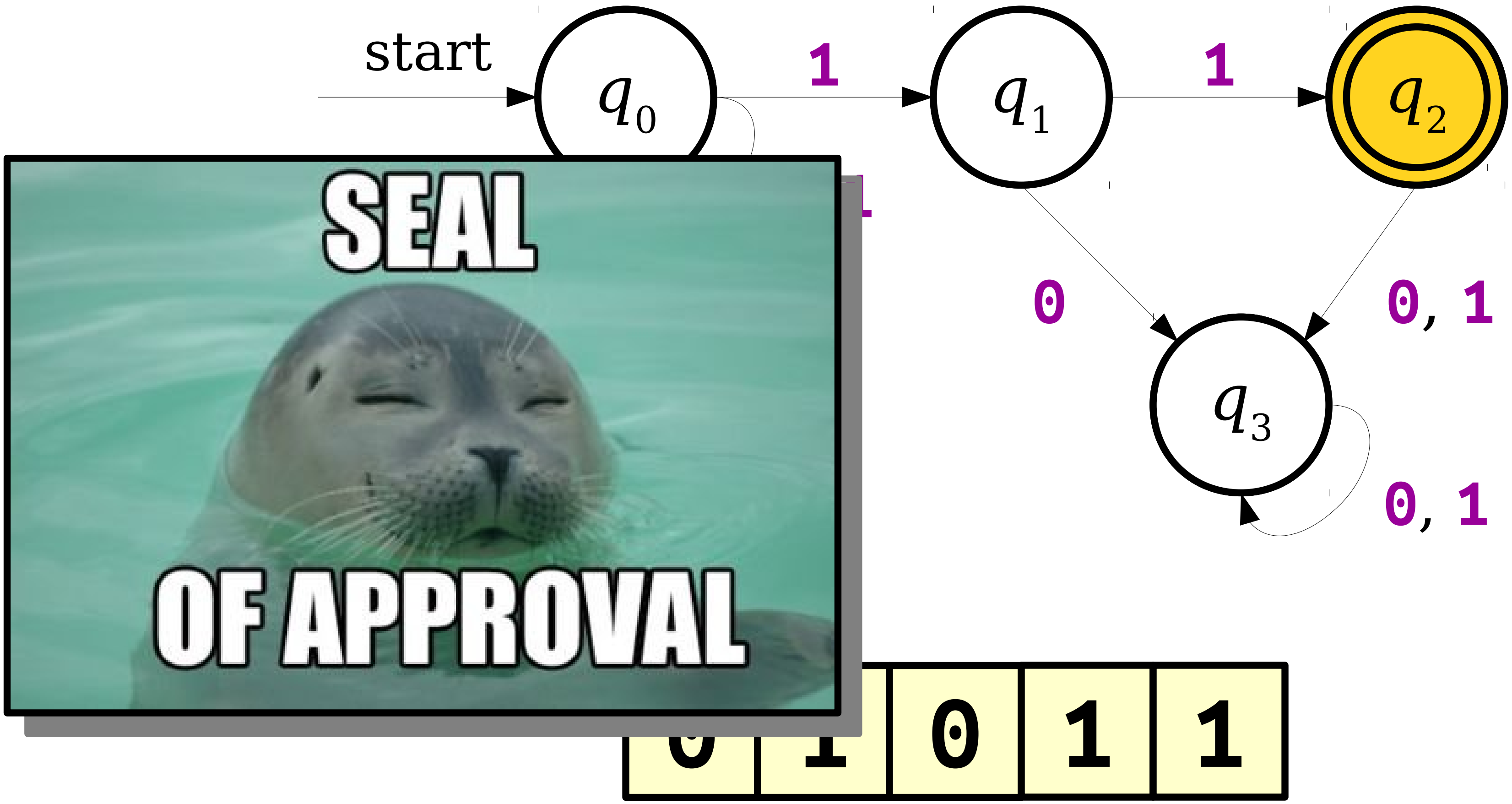
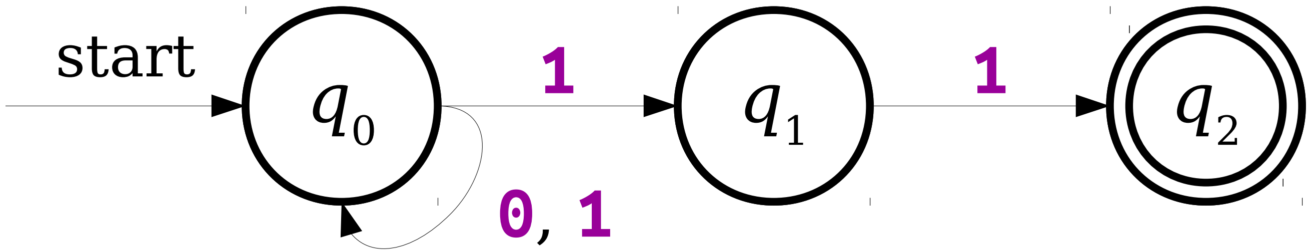


A More Complex NFA



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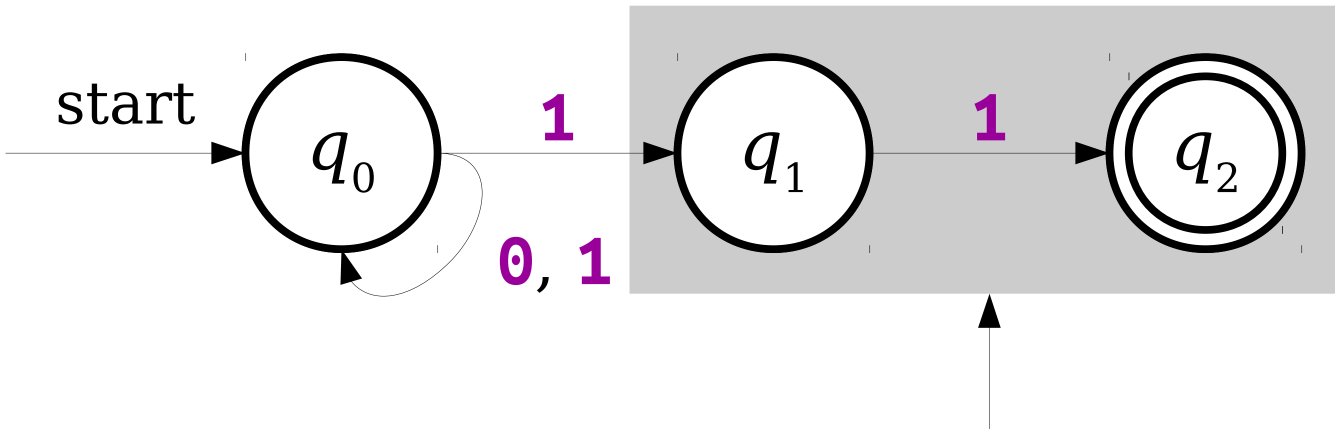
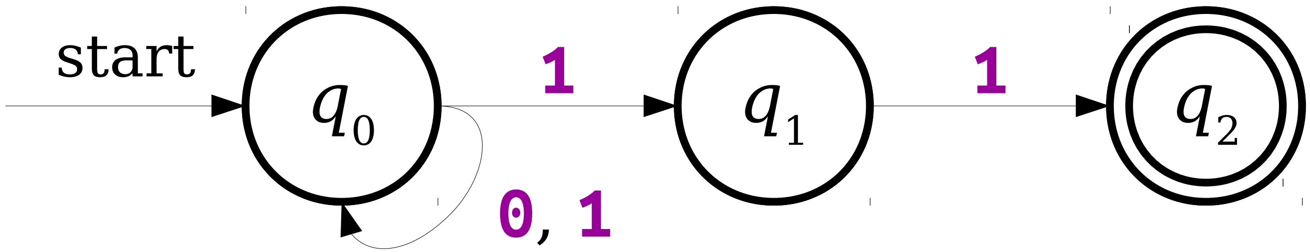
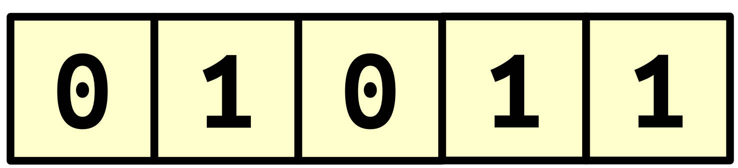


If a NFA needs to make a transition when no transition exists, the automaton dies and that particular path does not accept.

A More Complex NFA





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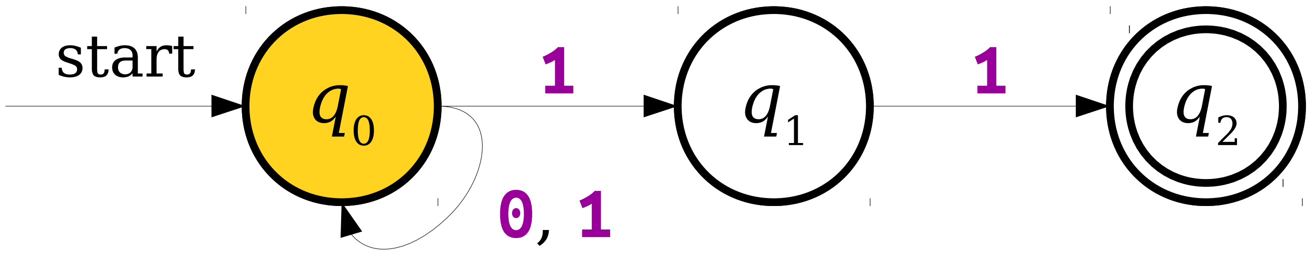

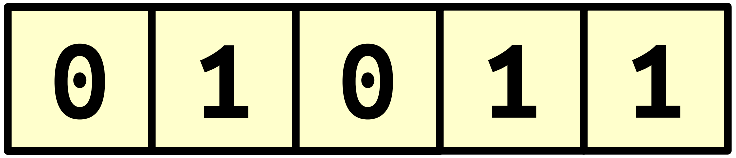

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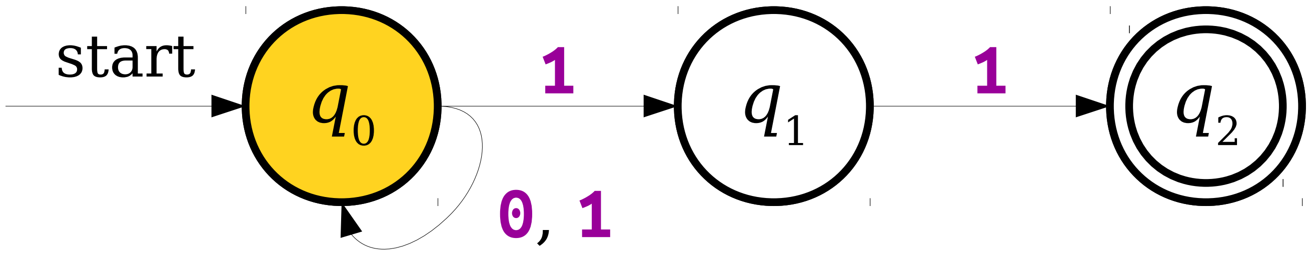

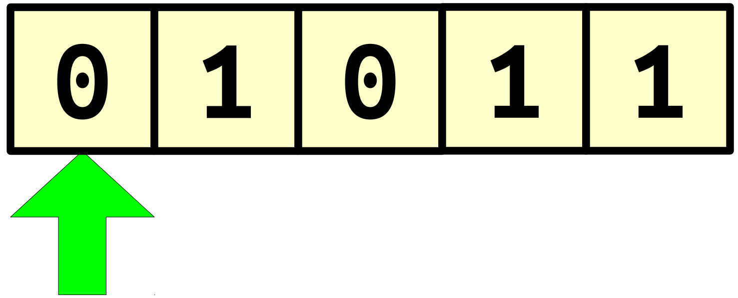

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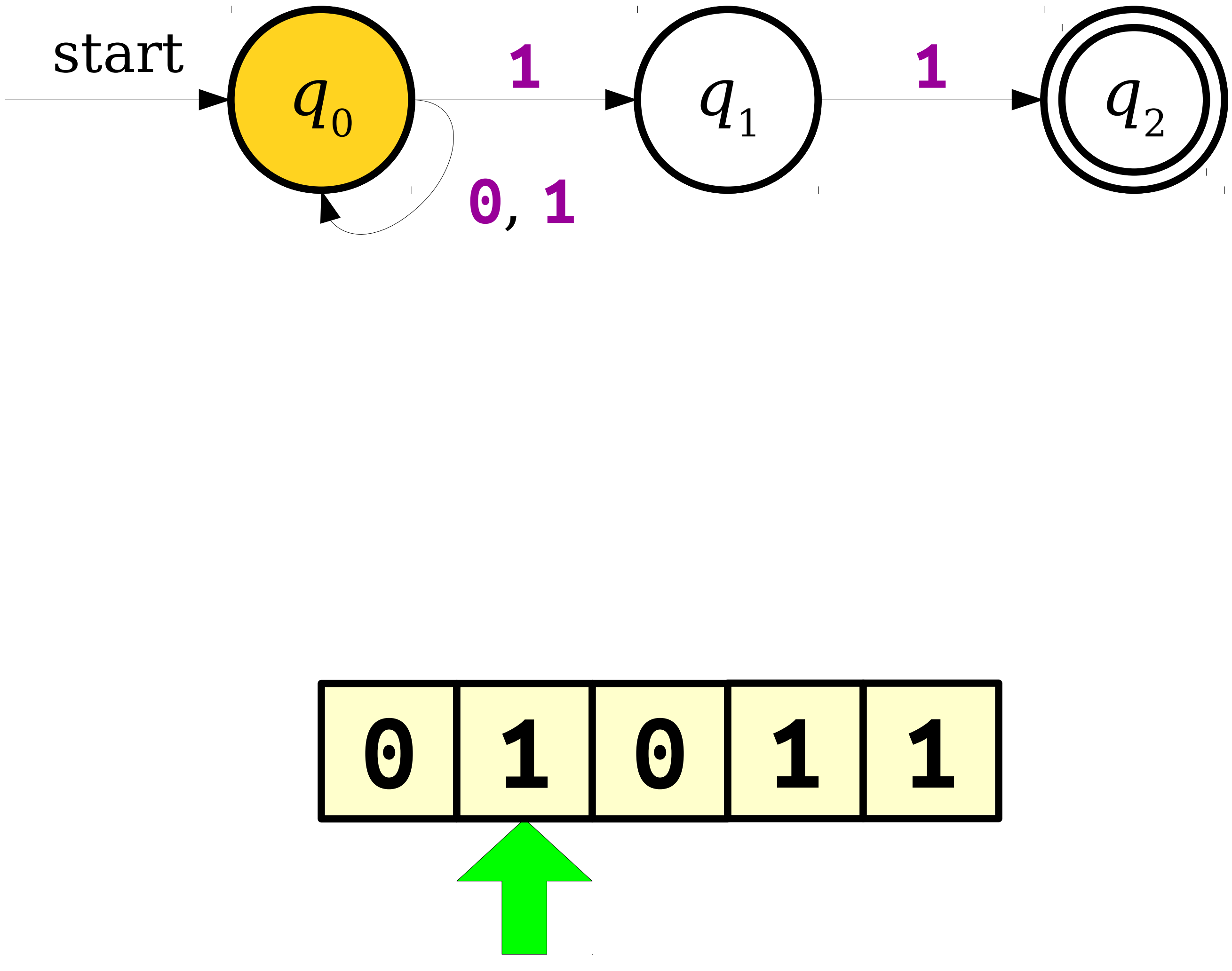

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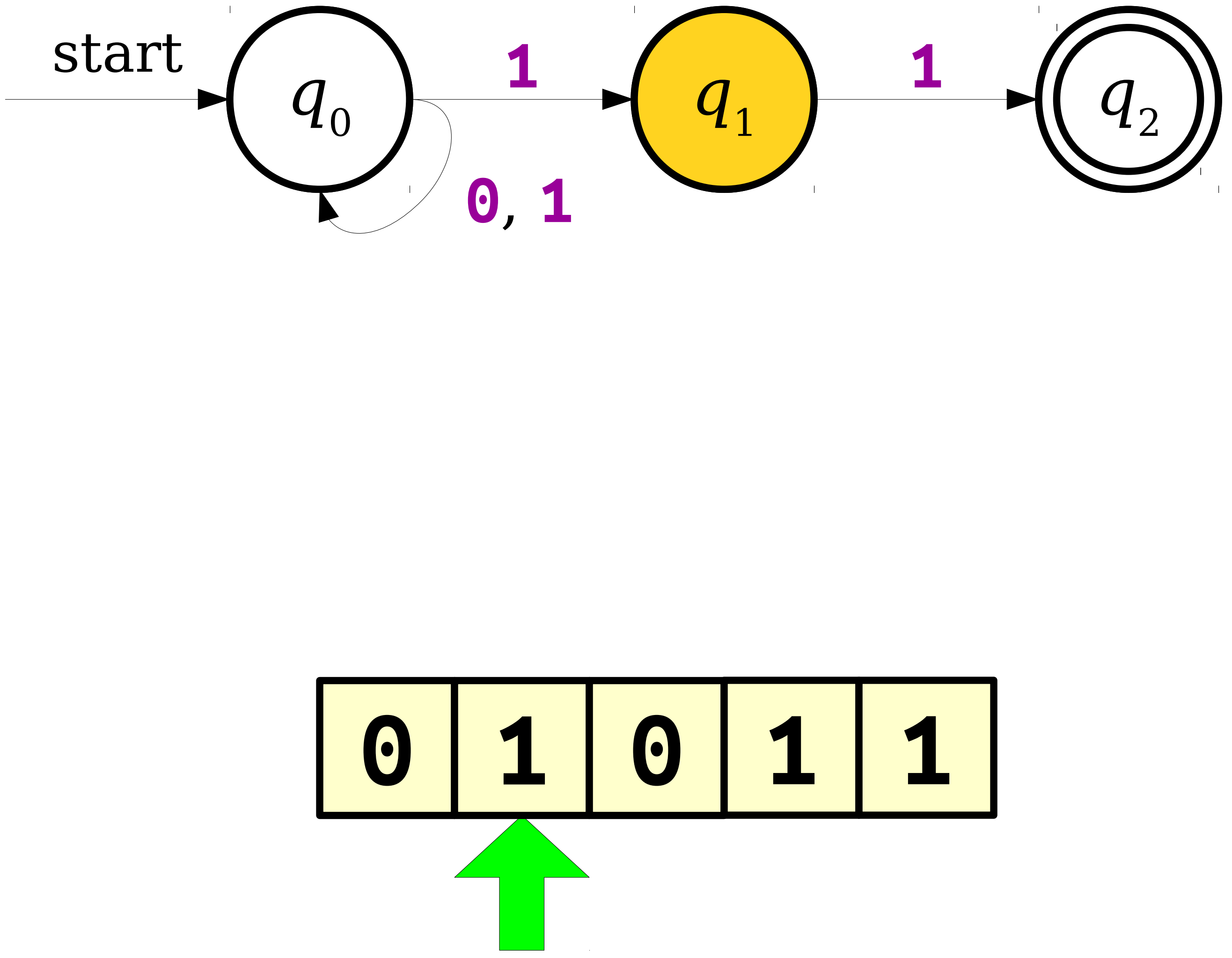

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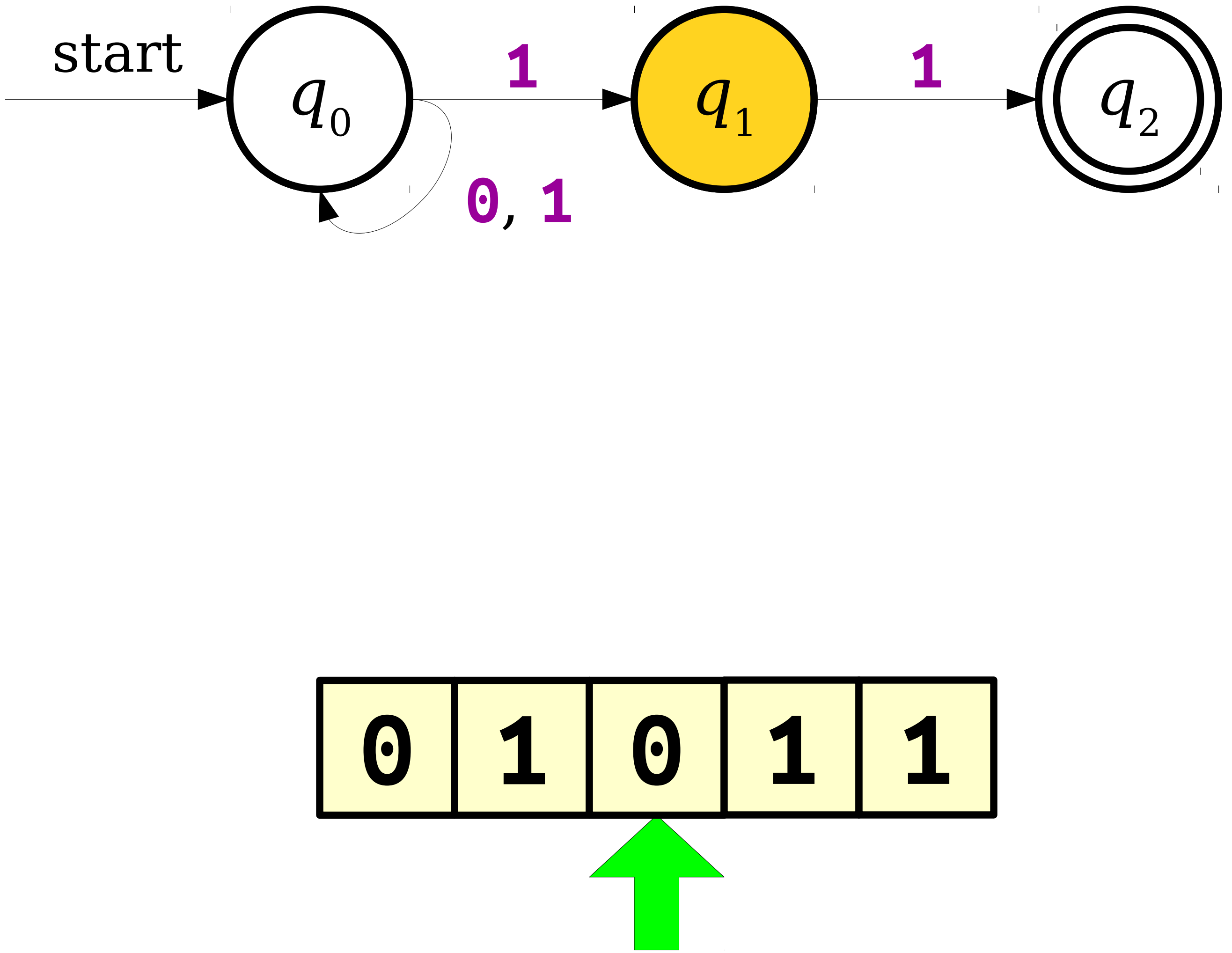

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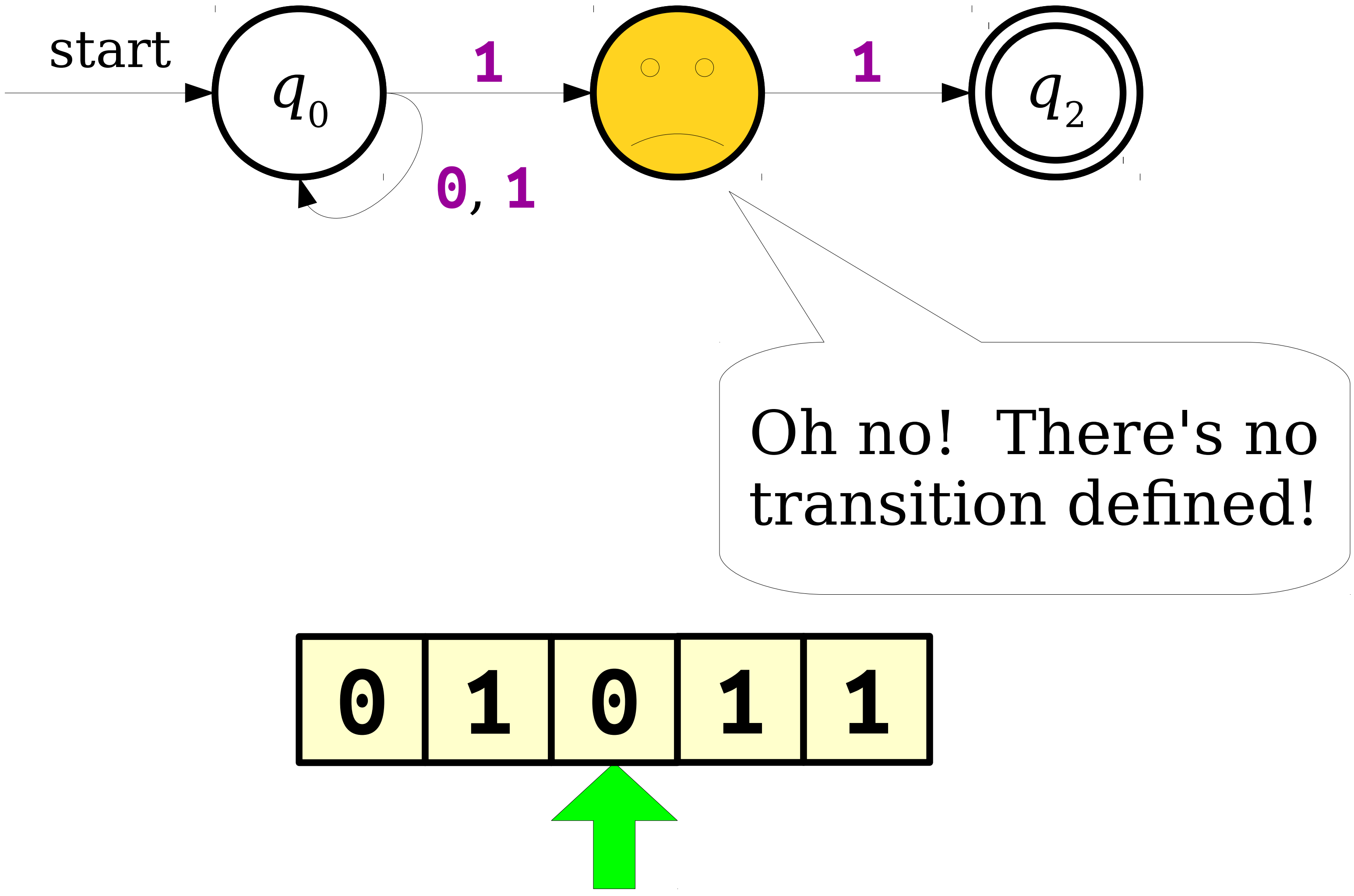

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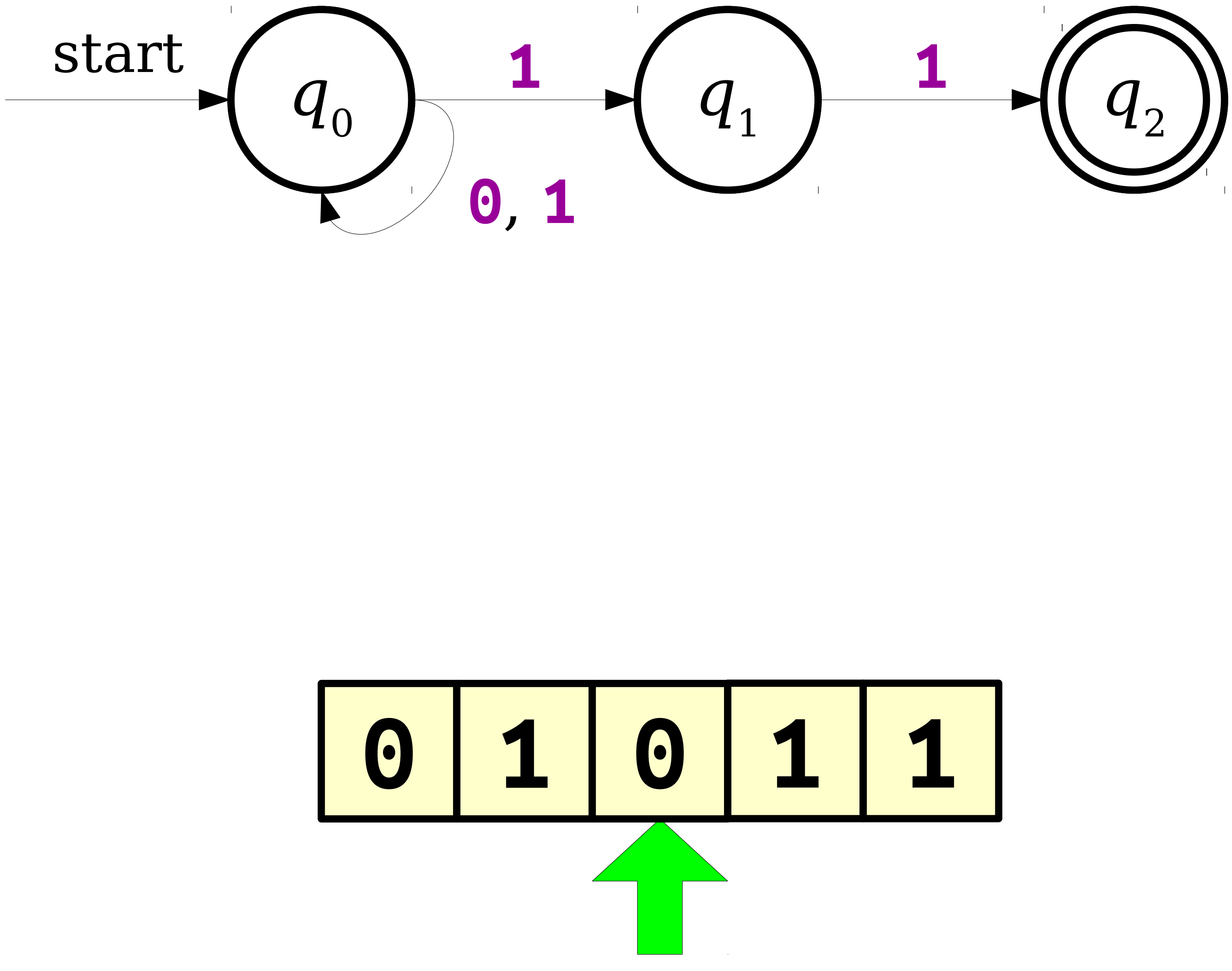

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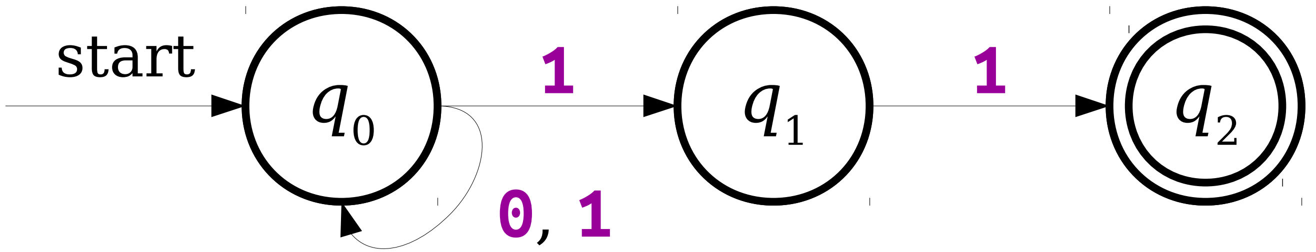

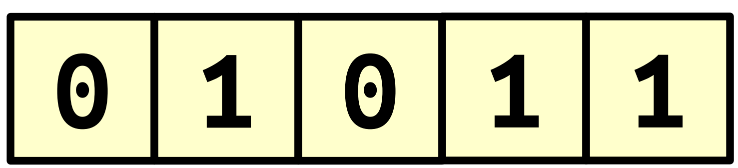

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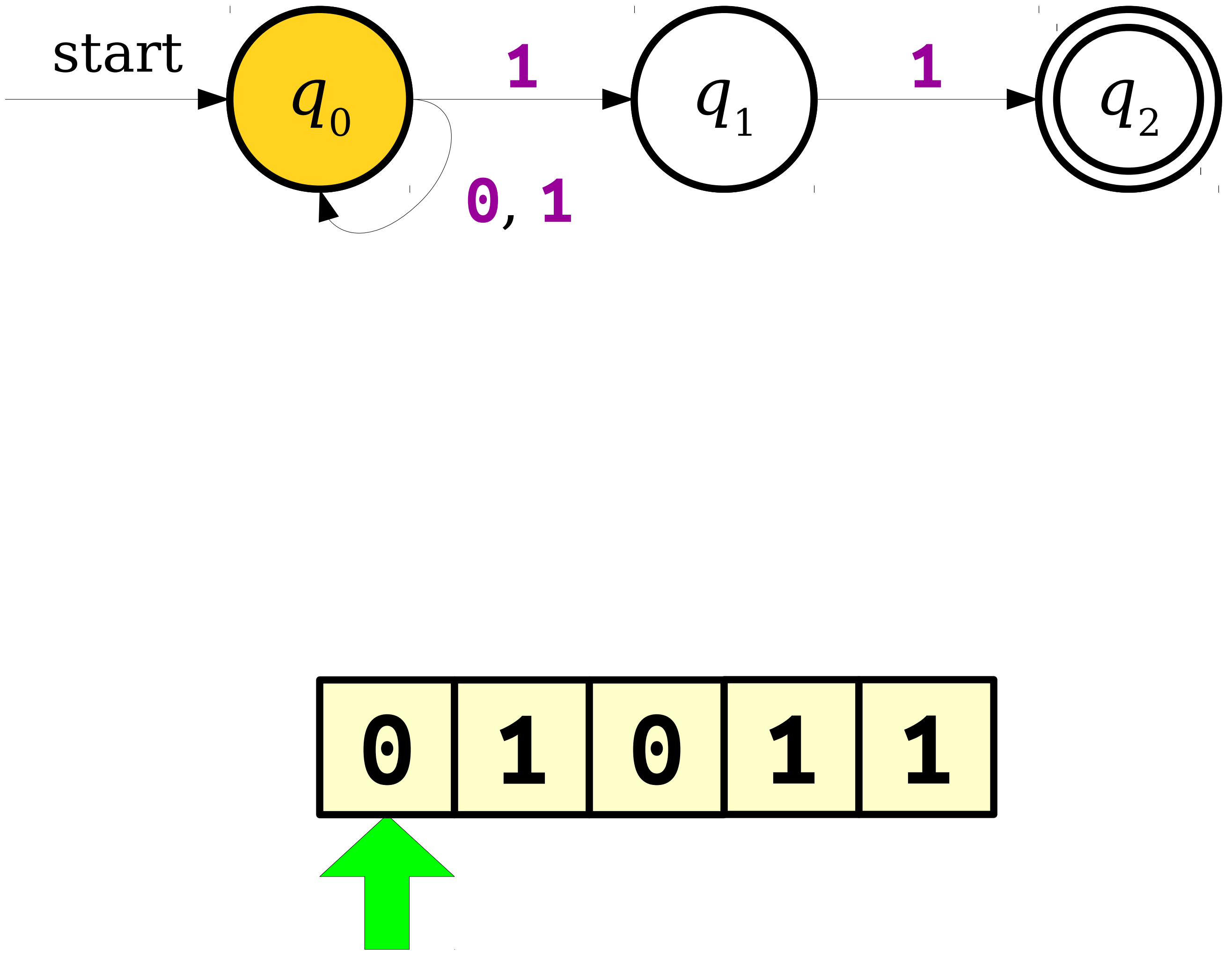

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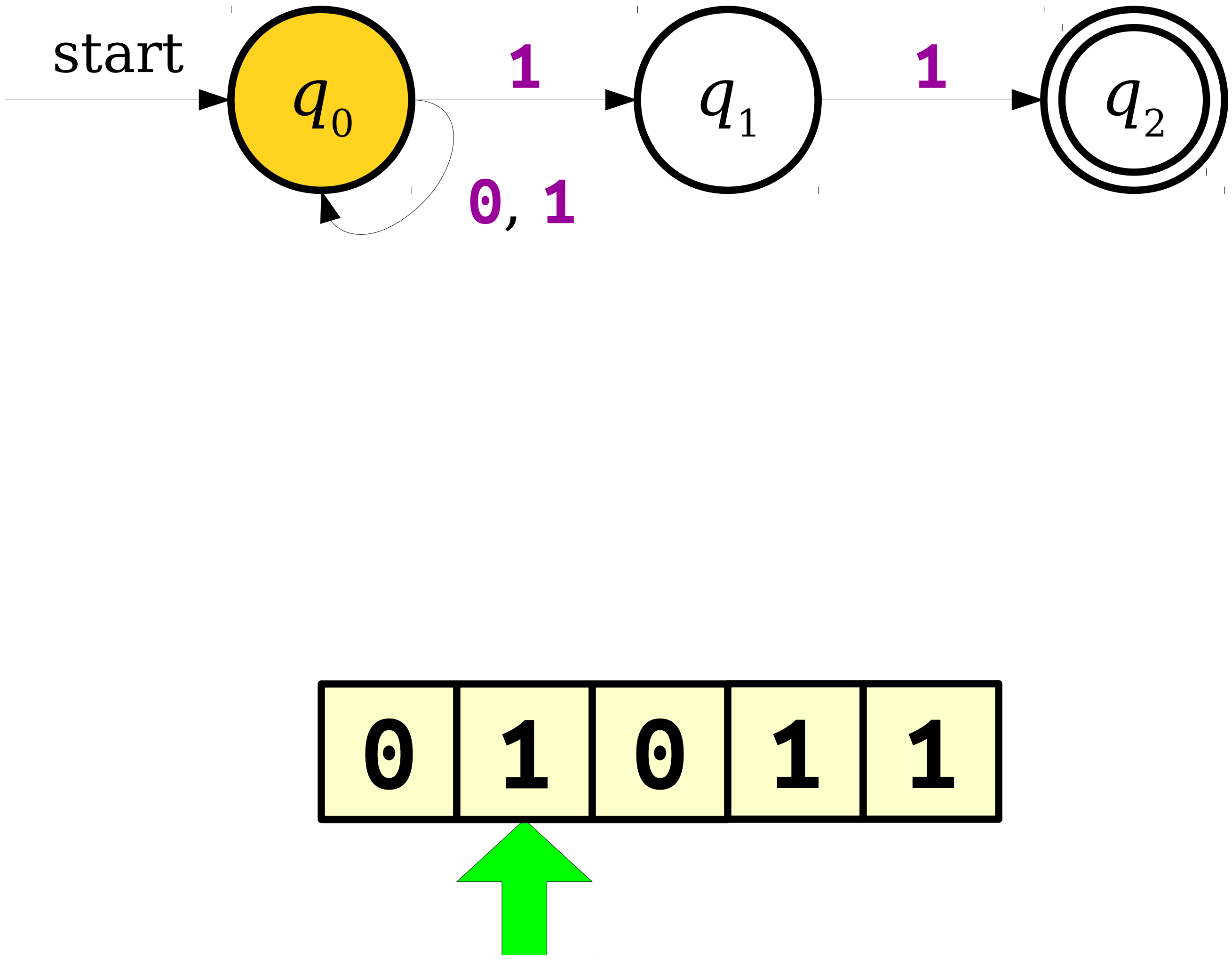

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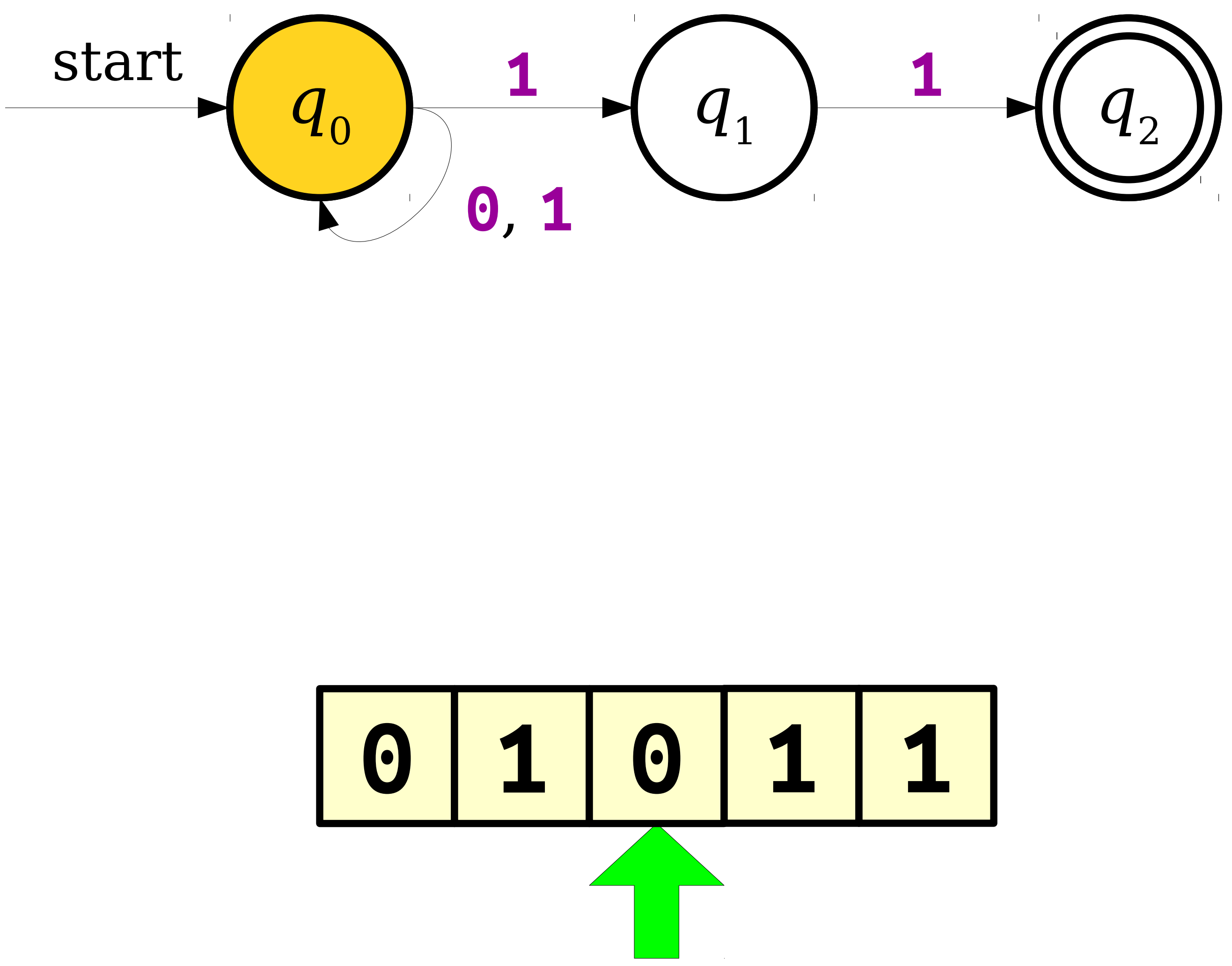

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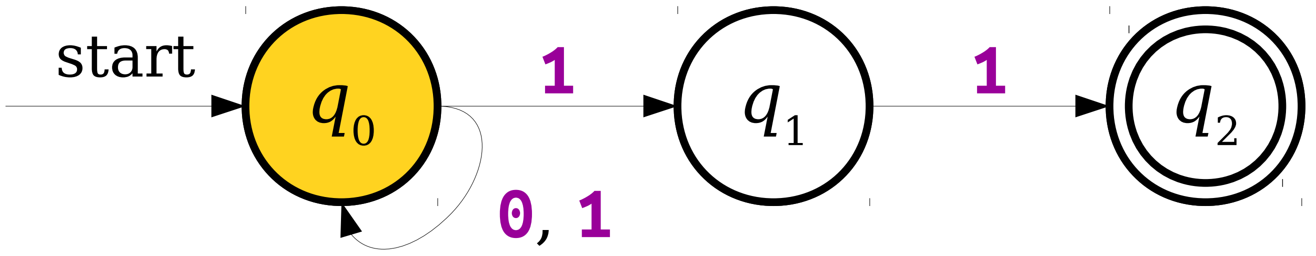

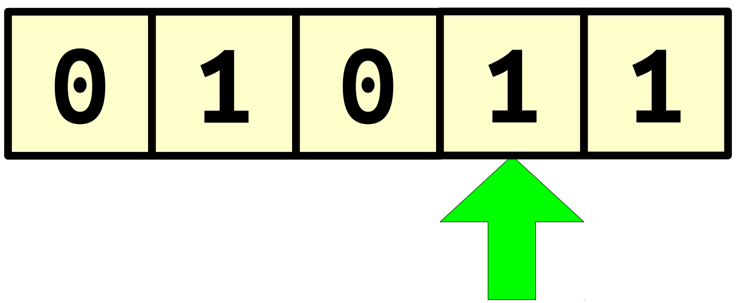

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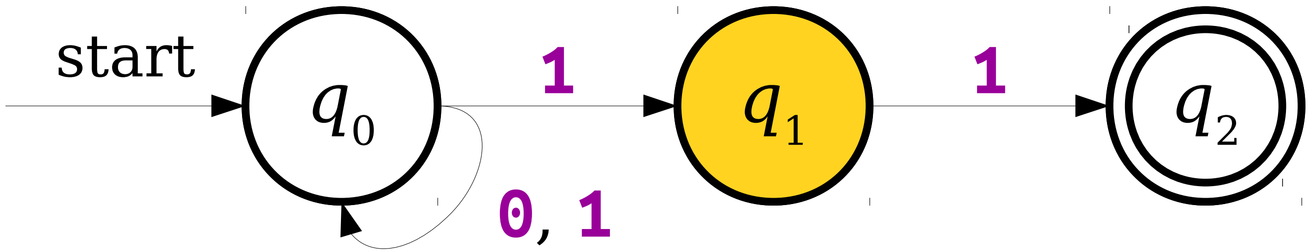

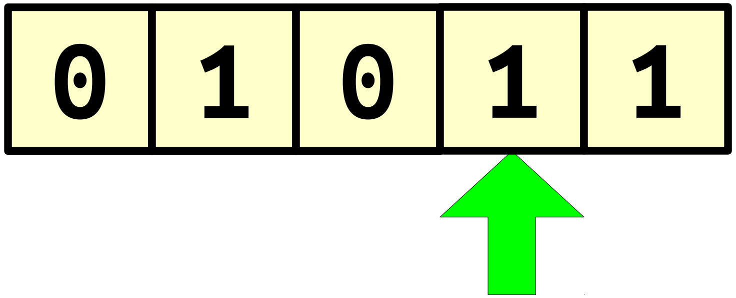

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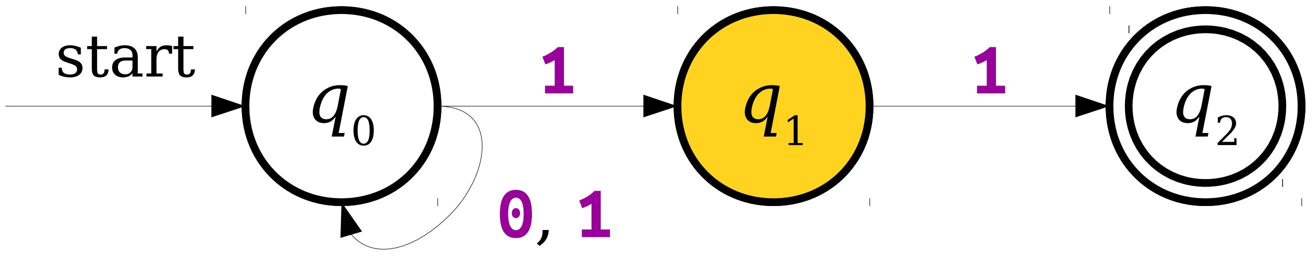

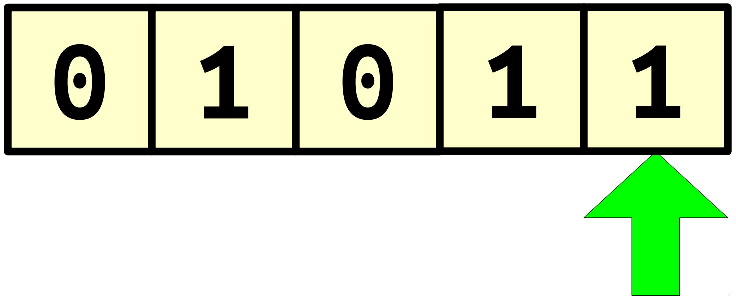

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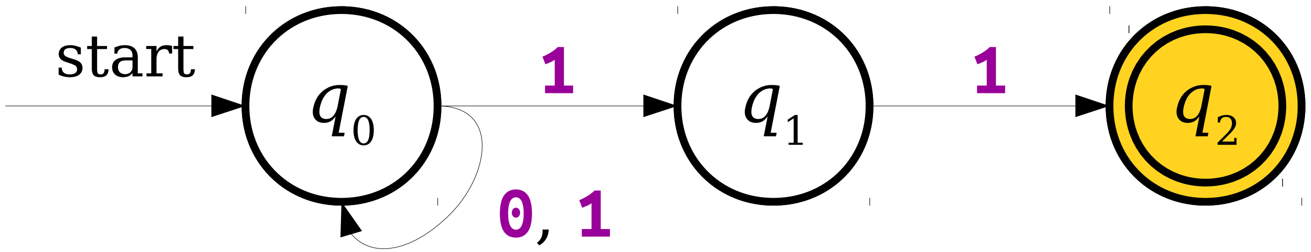

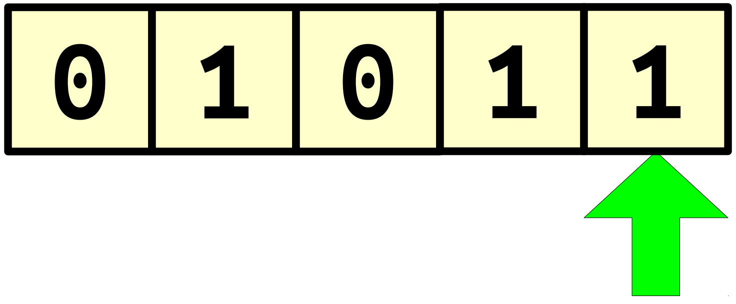

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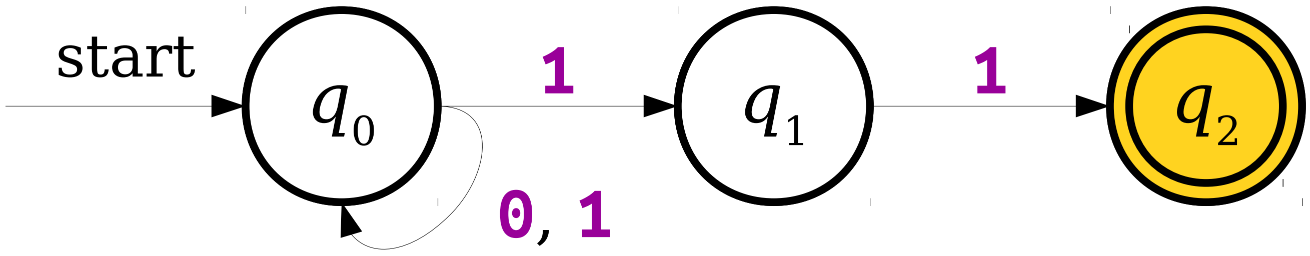

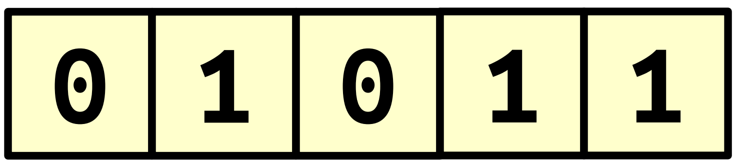

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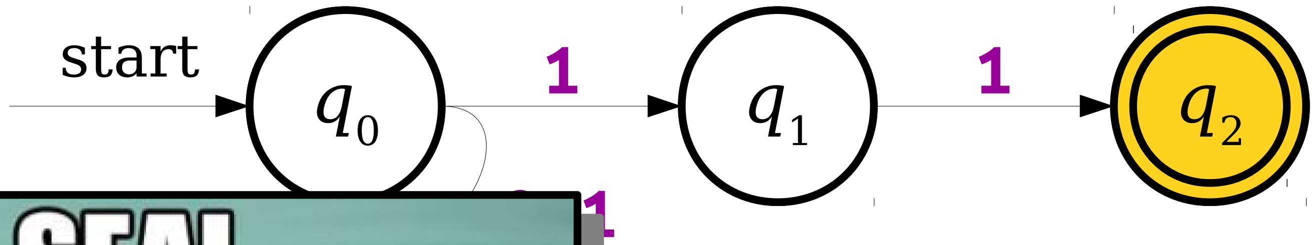

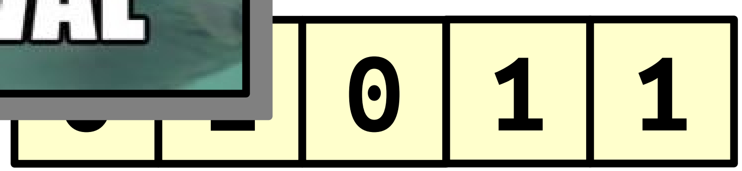

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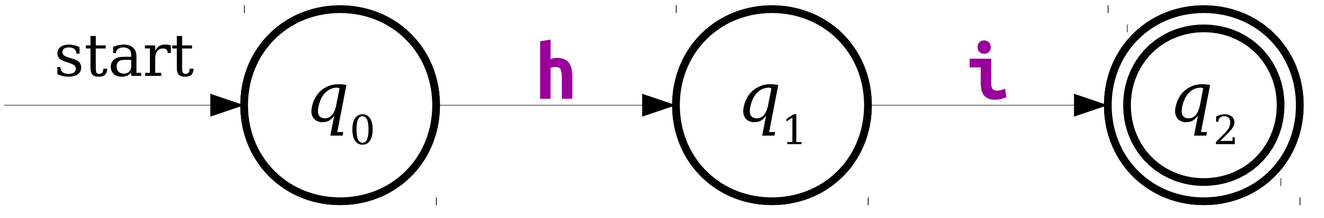

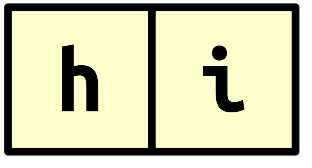

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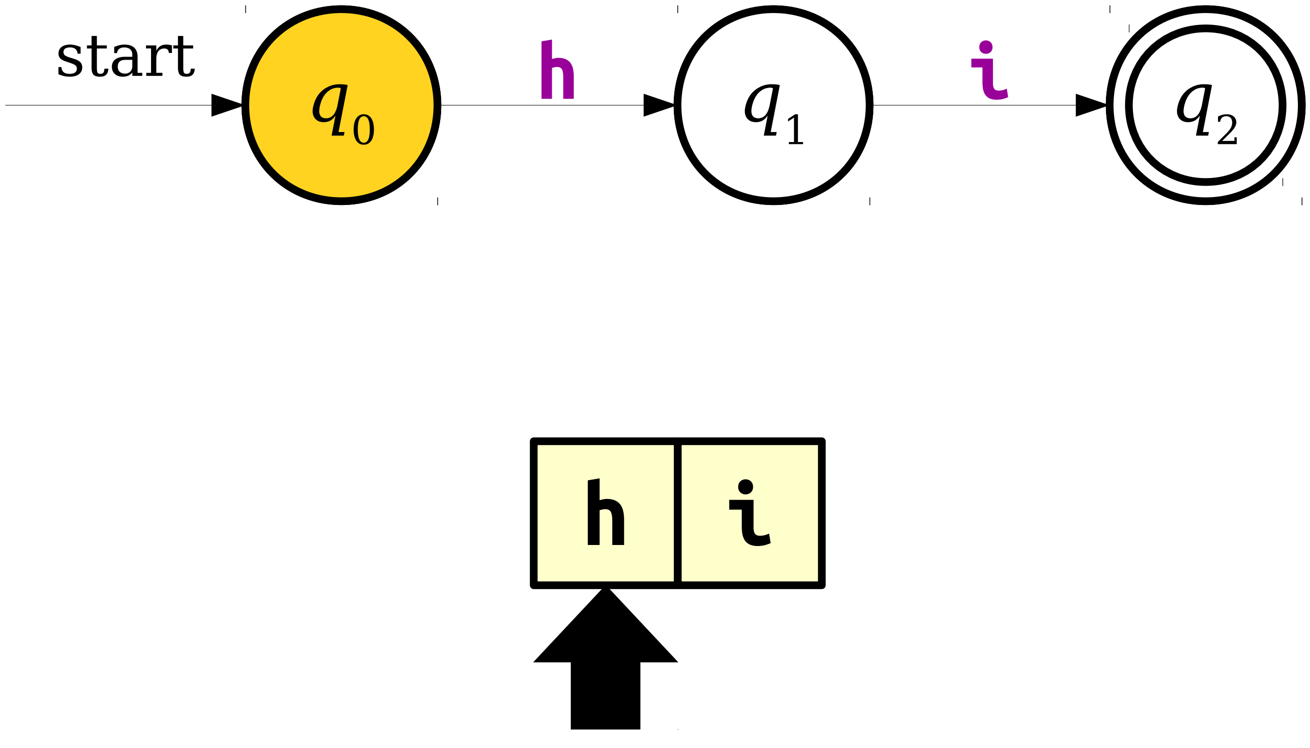

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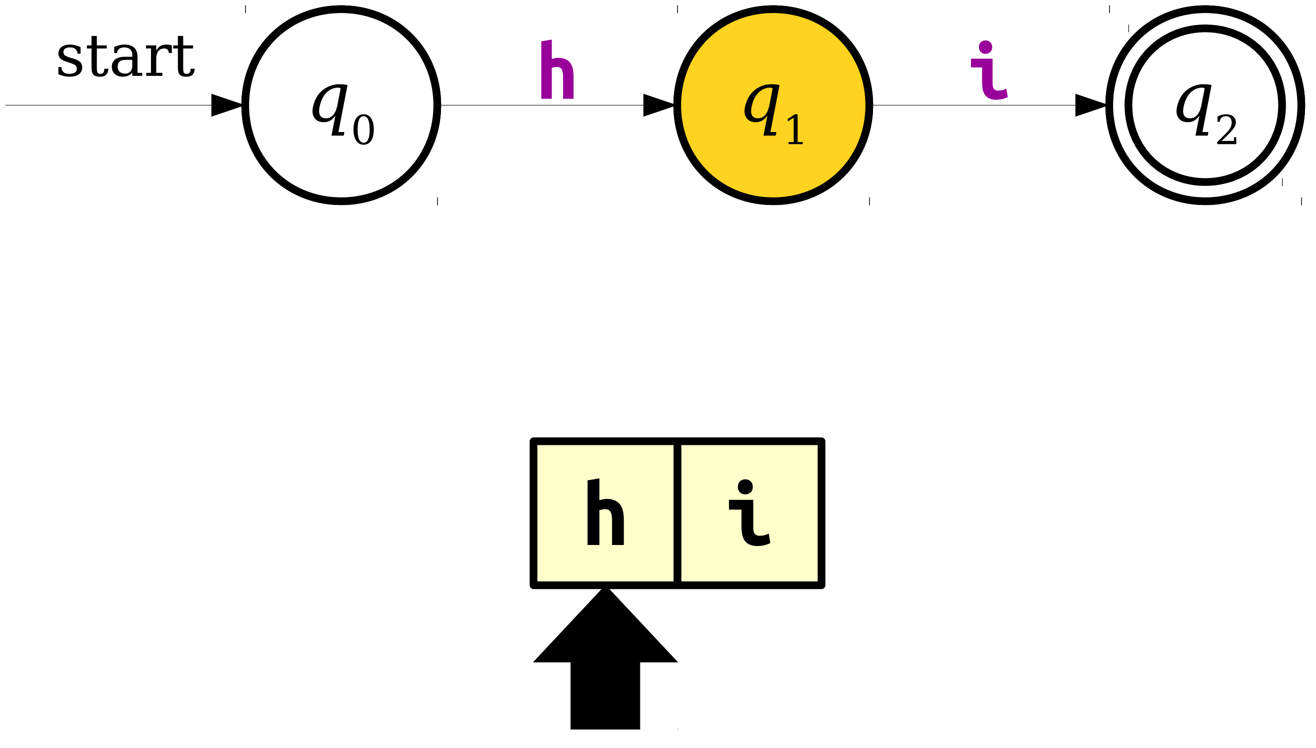

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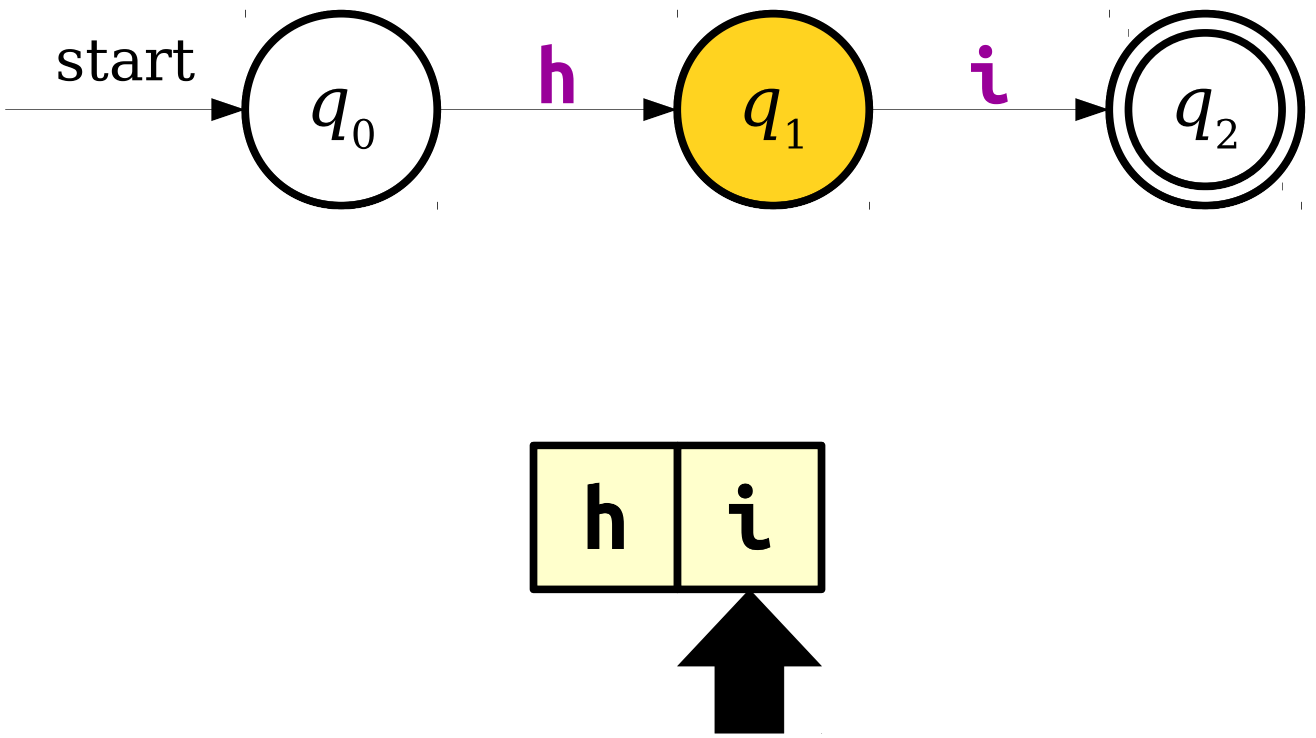

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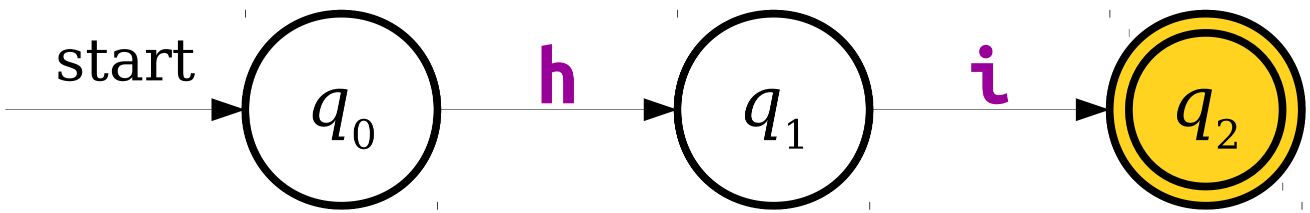

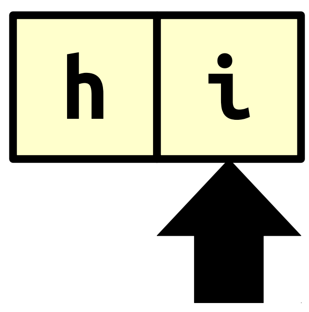

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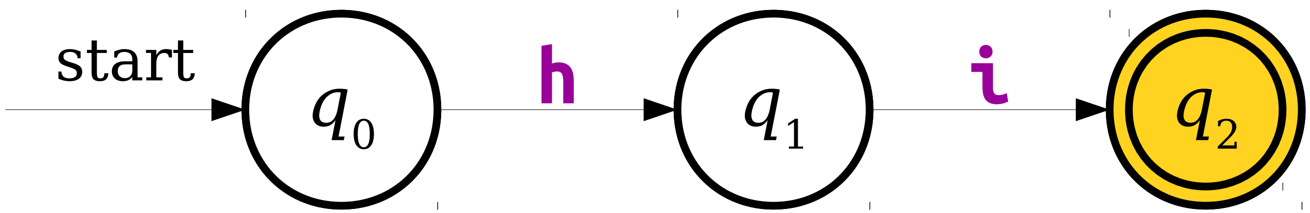
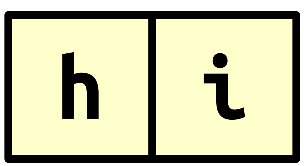
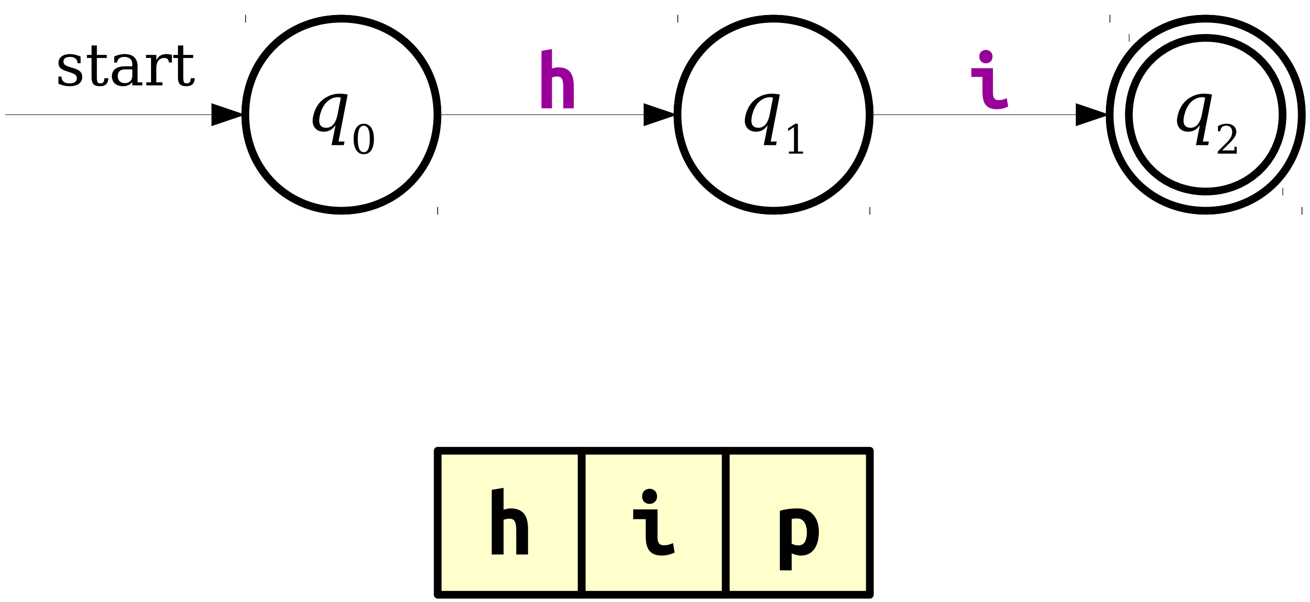




Tragedy in Paradise



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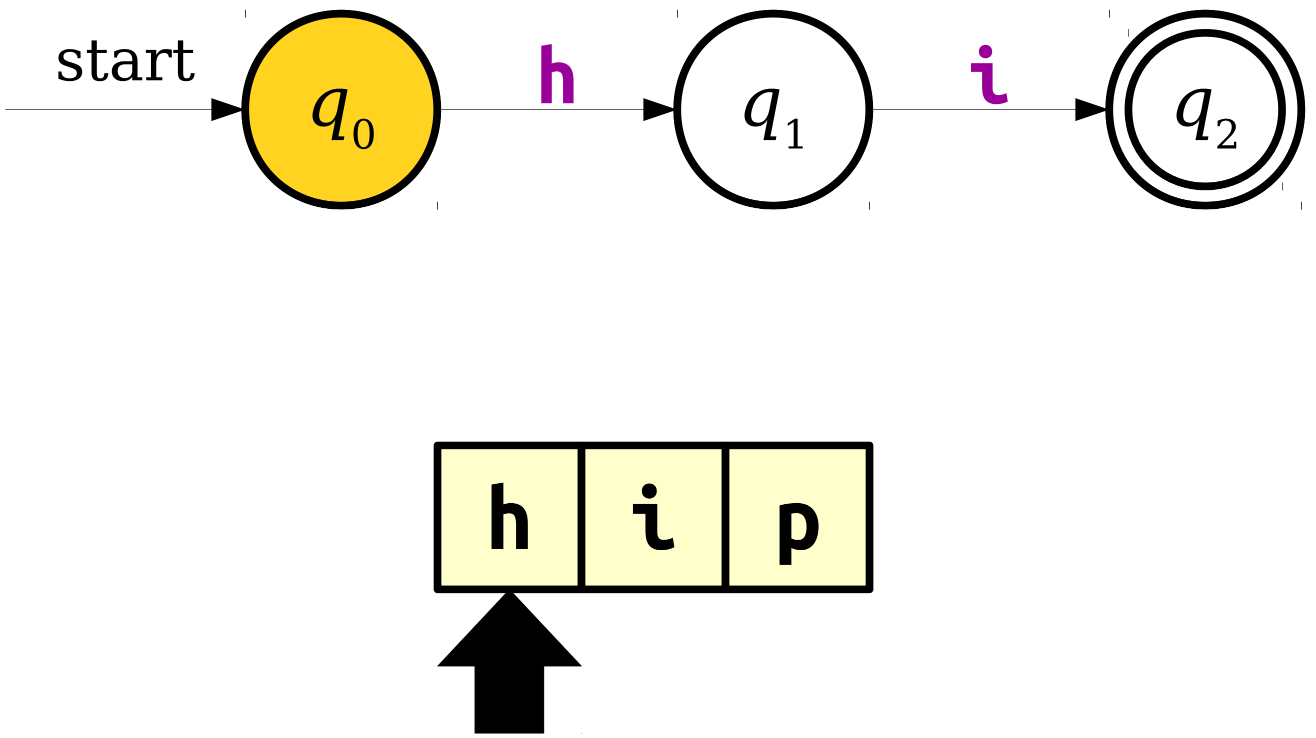

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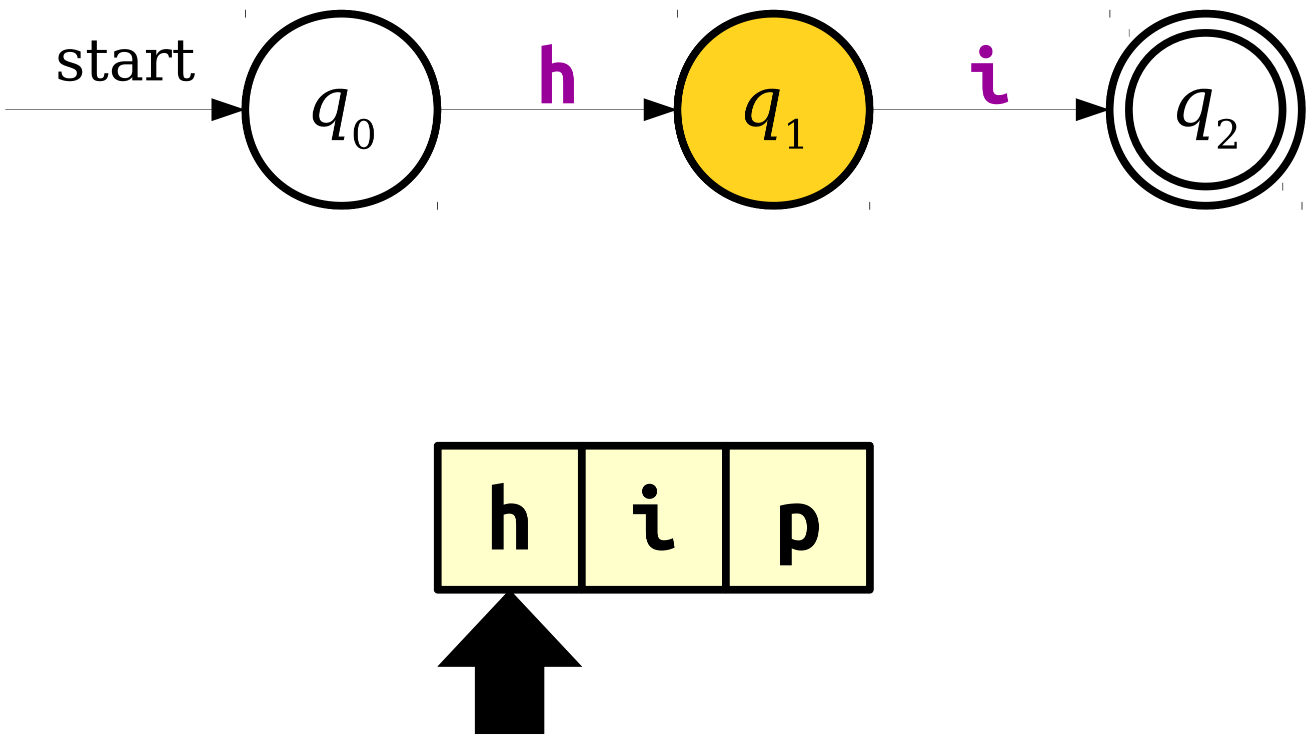

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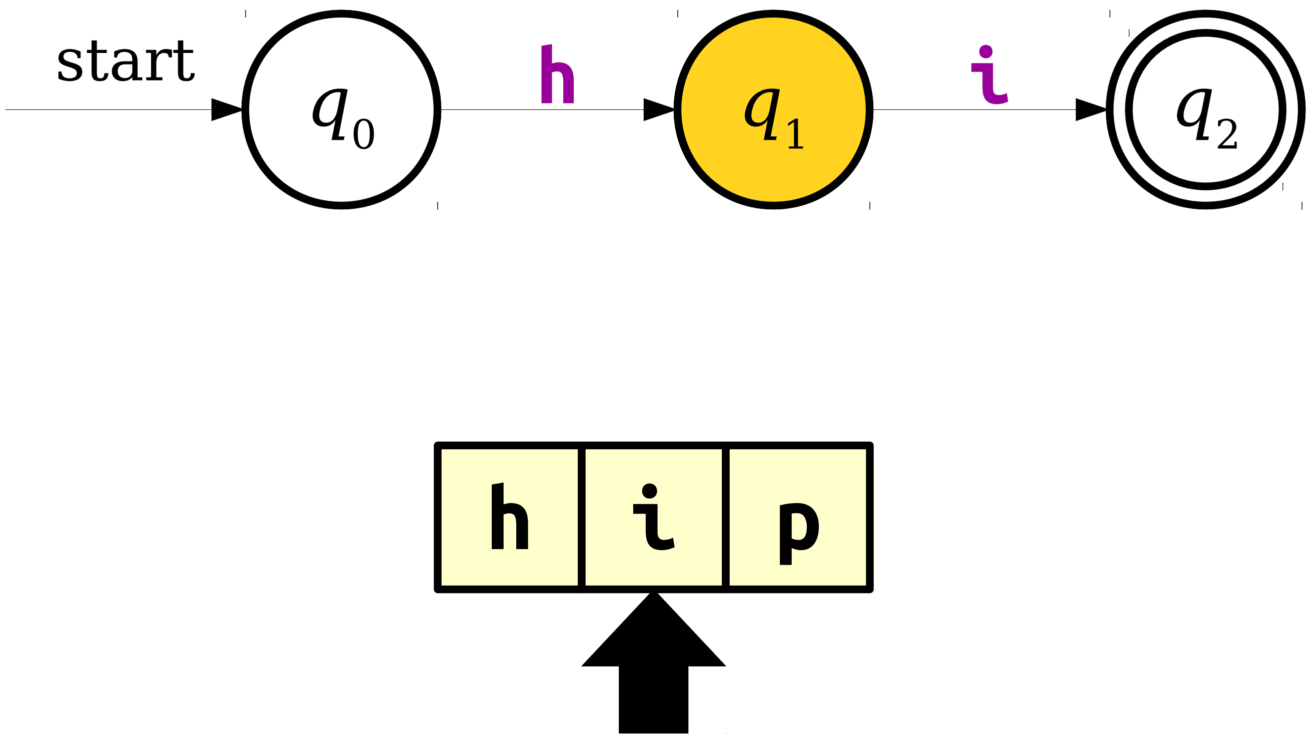

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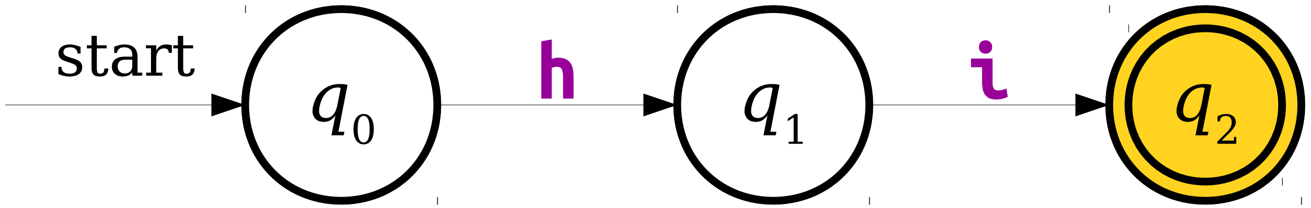

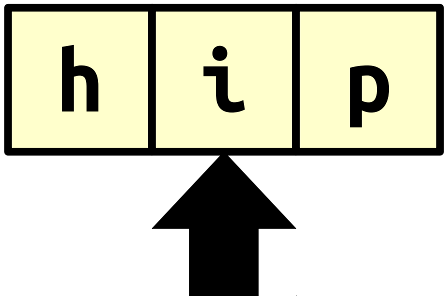

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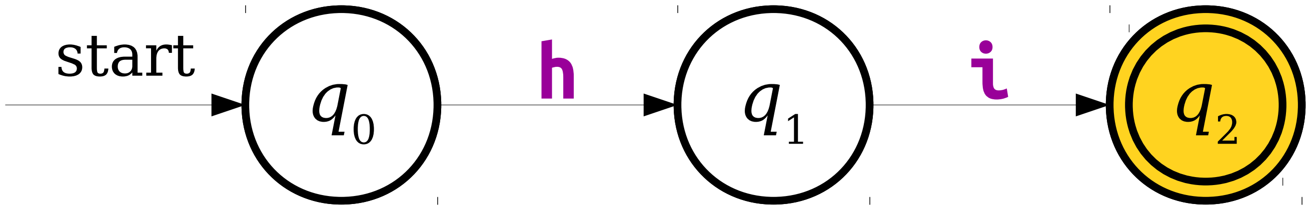

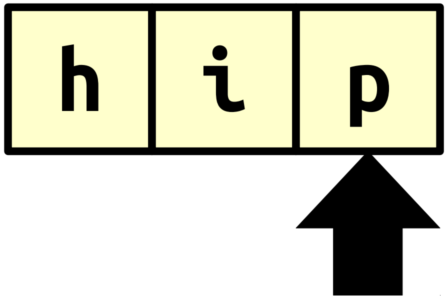

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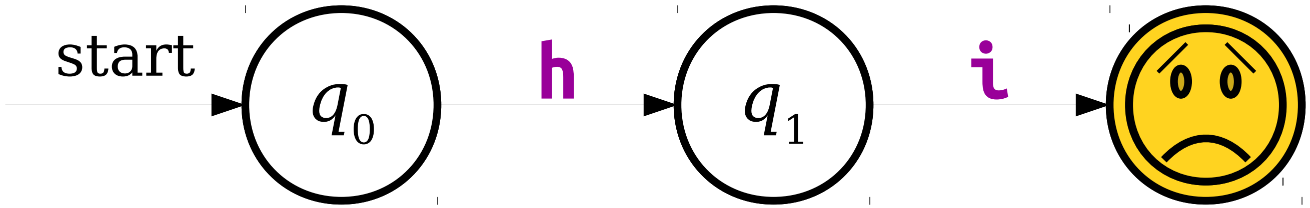

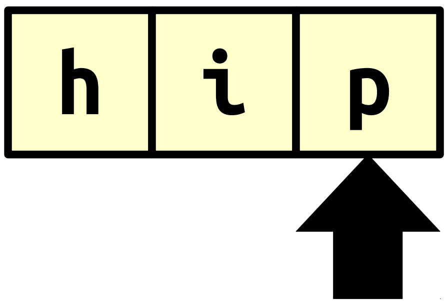

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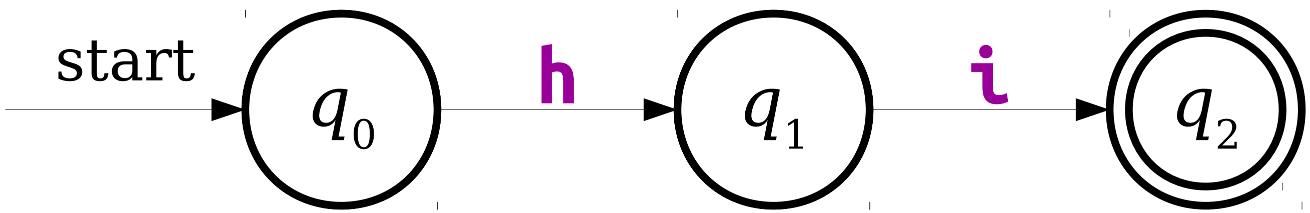

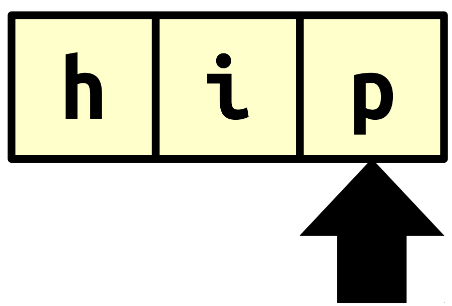

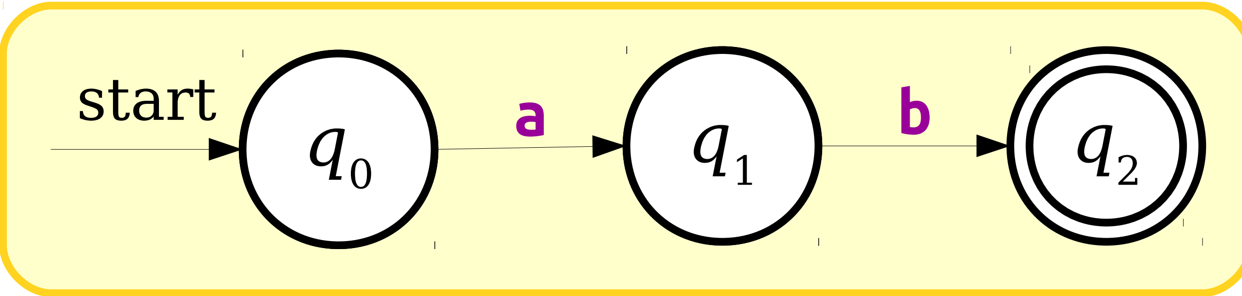

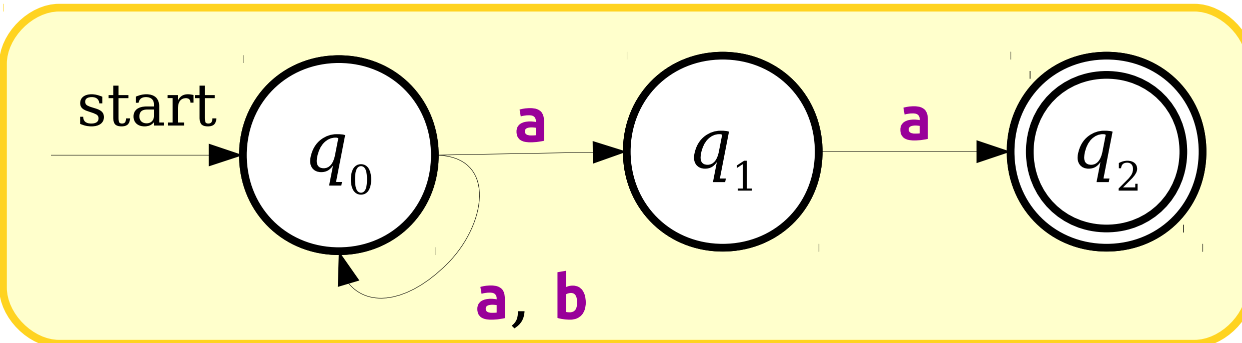

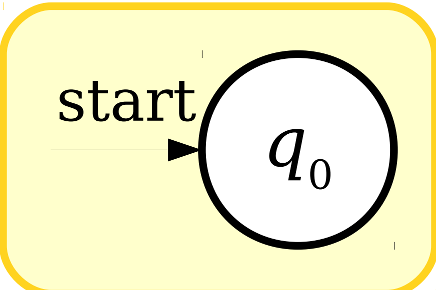

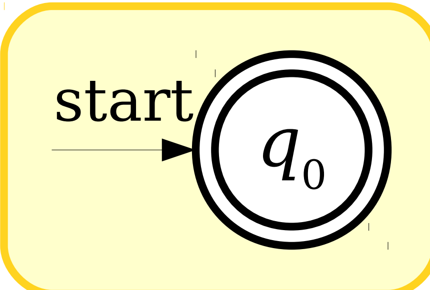

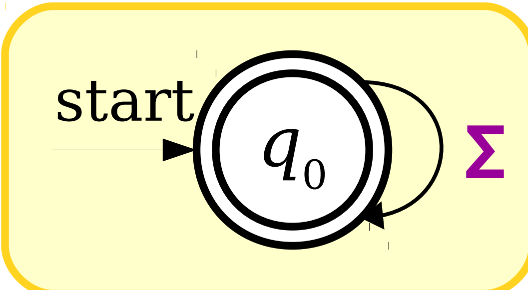

The ***language of an NFA*** is

$$\mathcal{L}(N) = \{ w \in \Sigma^* \mid N \text{ accepts } w \}.$$

What is the language of each NFA? (Assume Σ = {a, b}.)
Answer at ***https://pollev.com/cs103***

ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.

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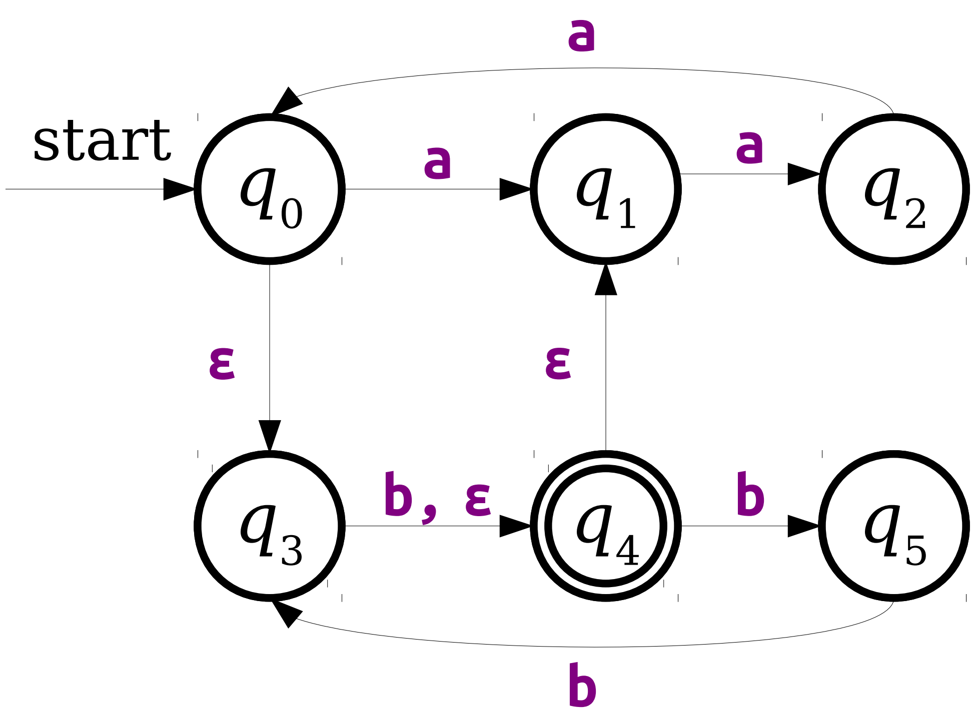

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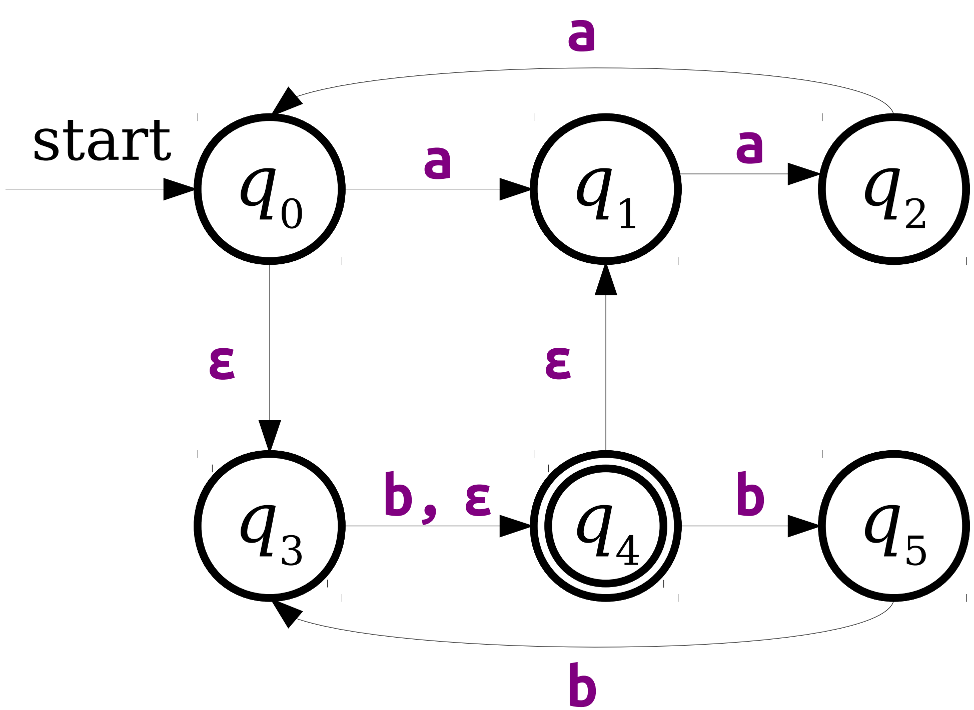

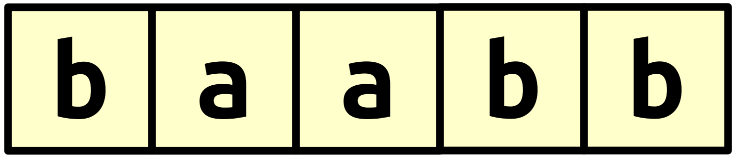

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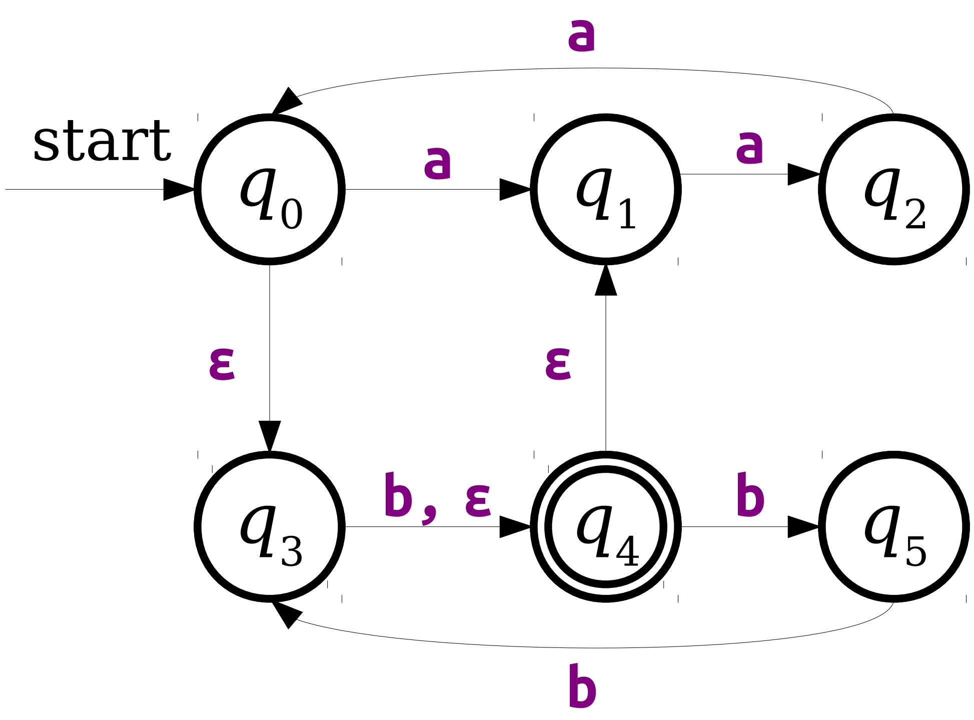

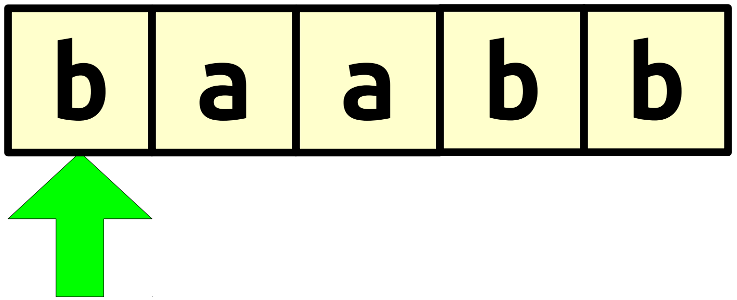

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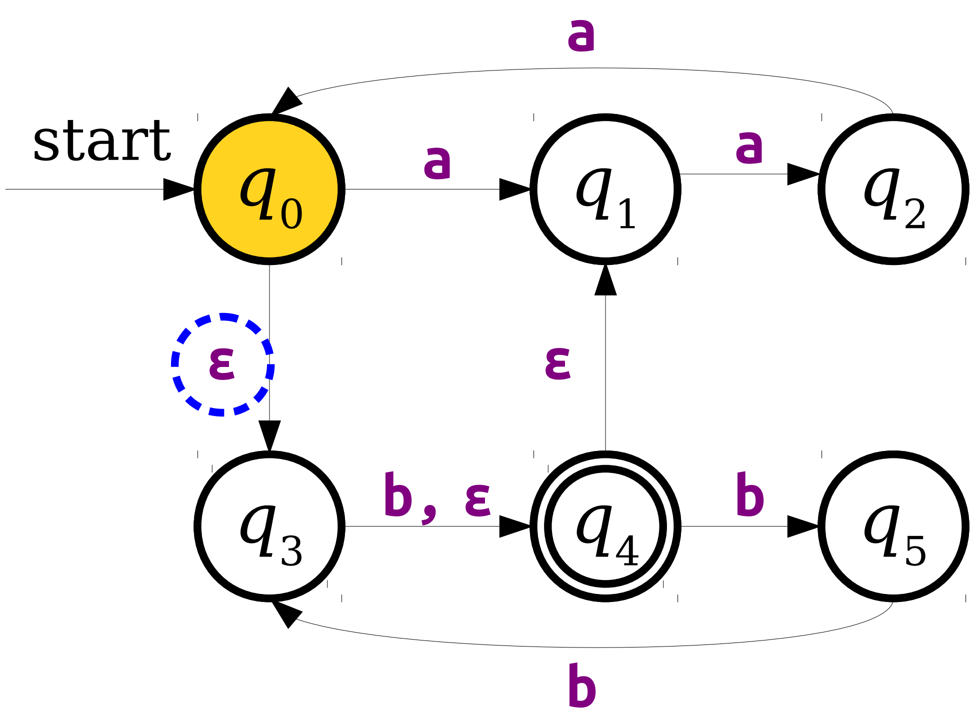

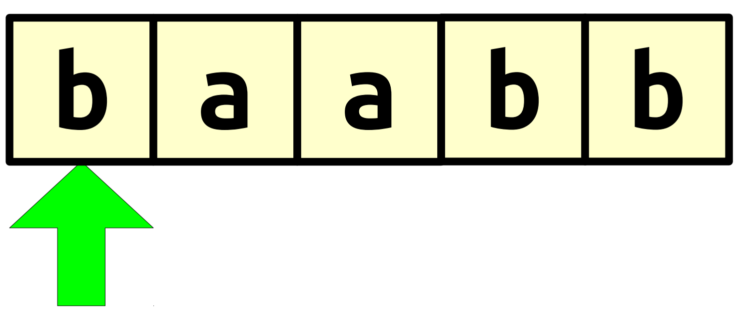

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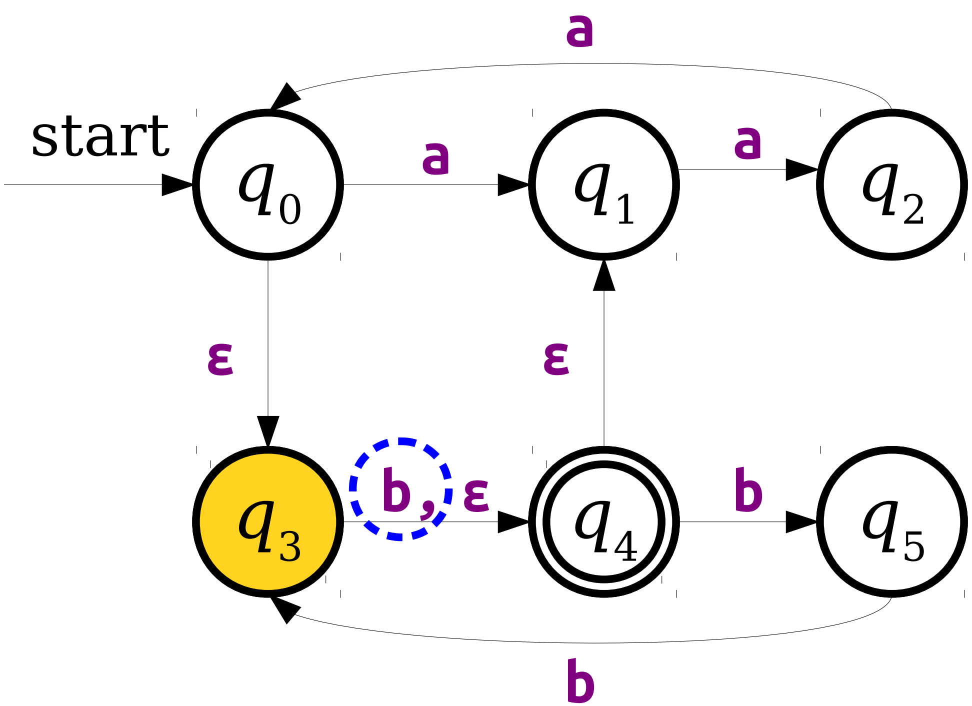

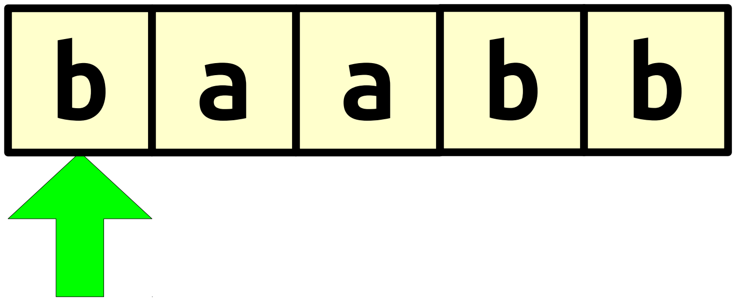

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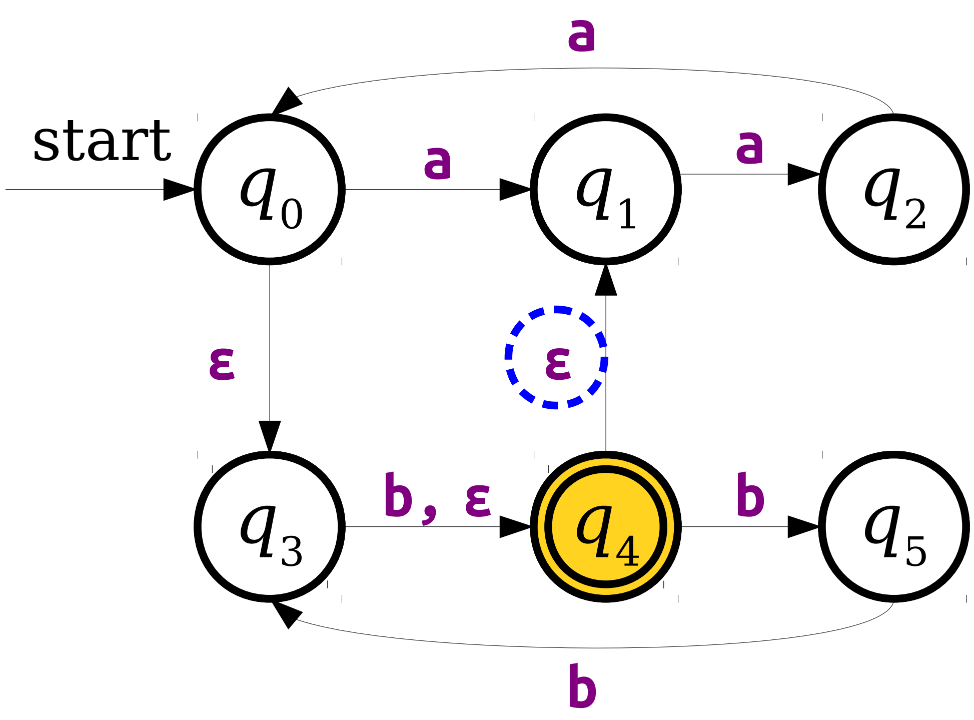

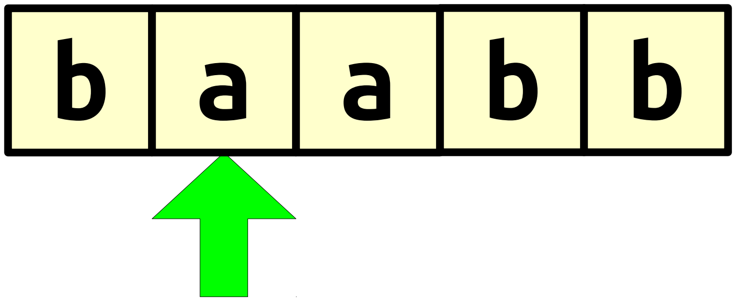

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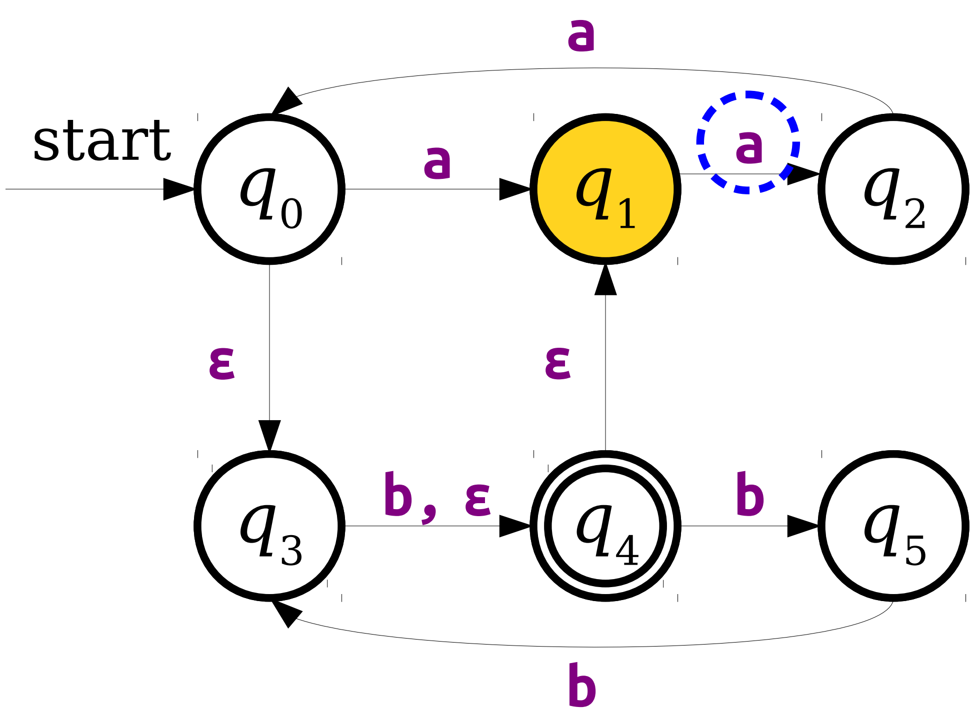

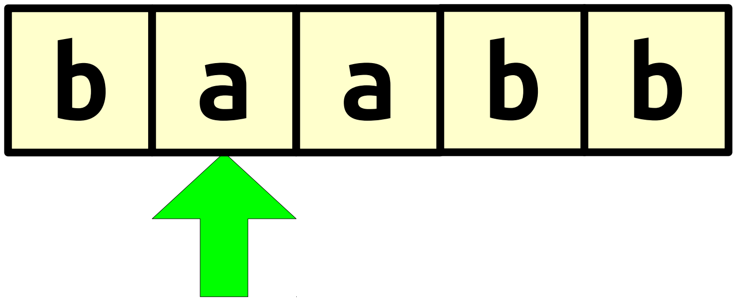

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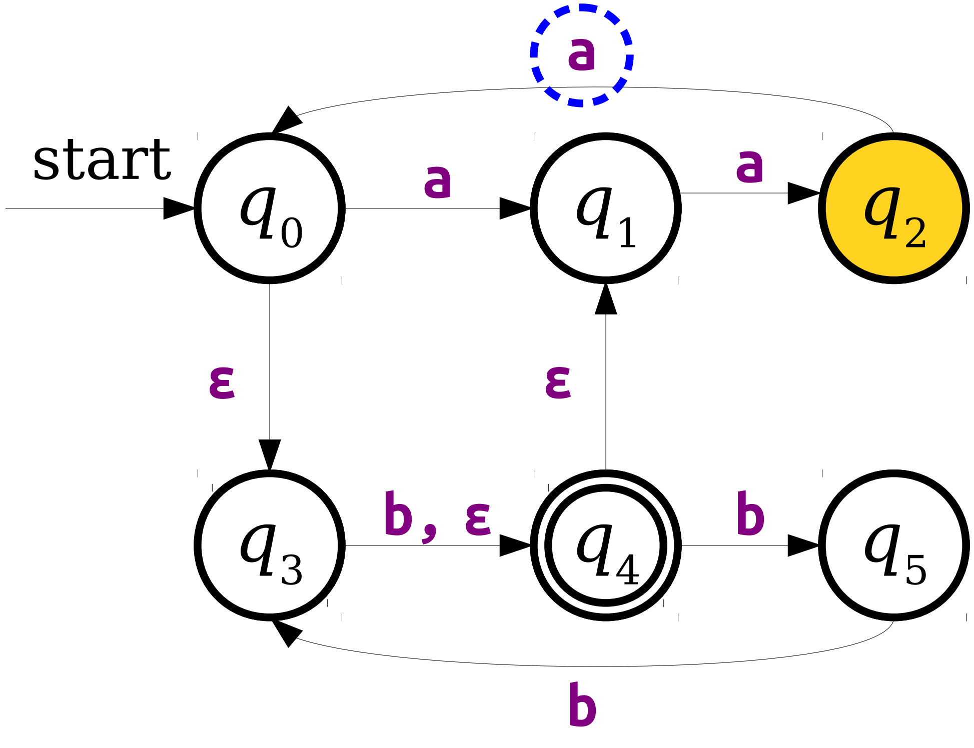

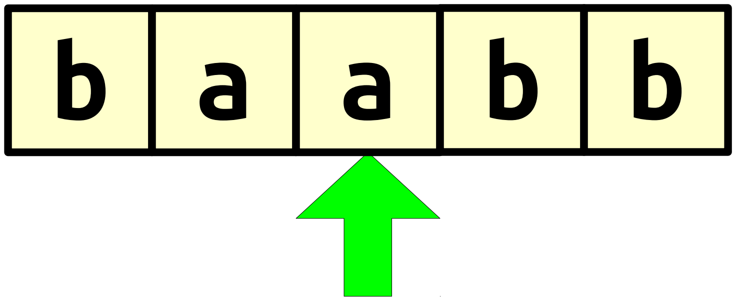

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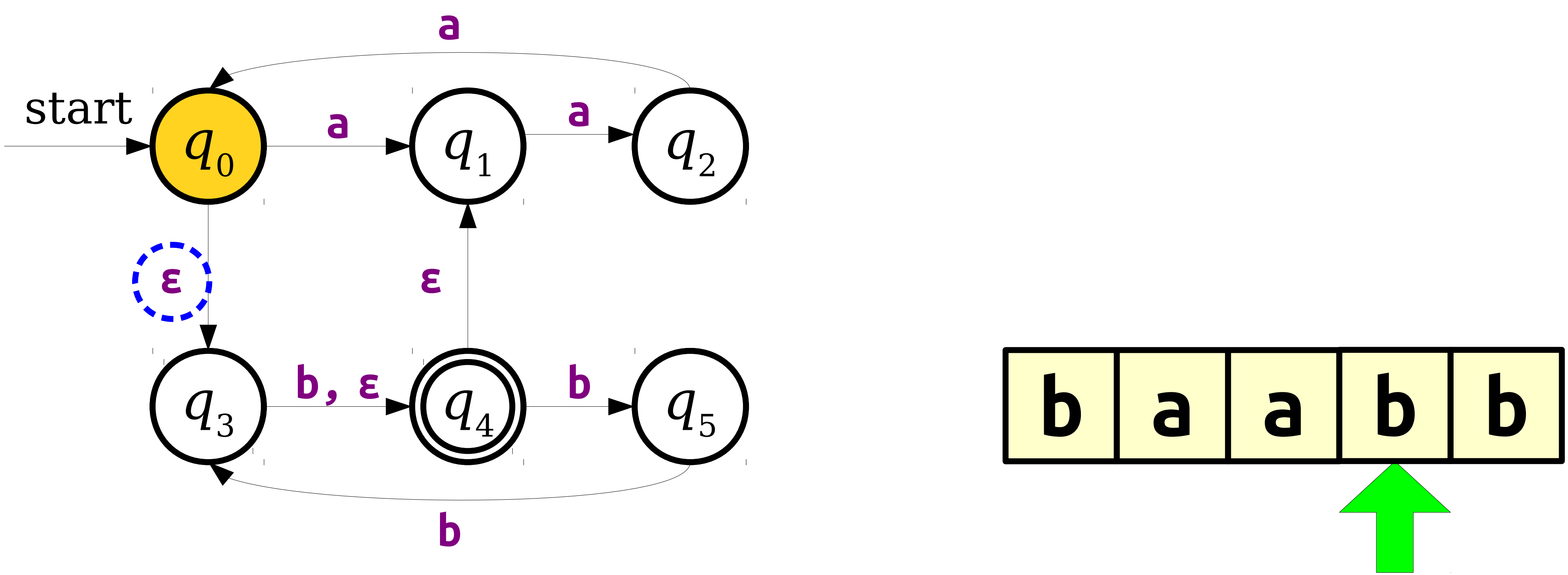

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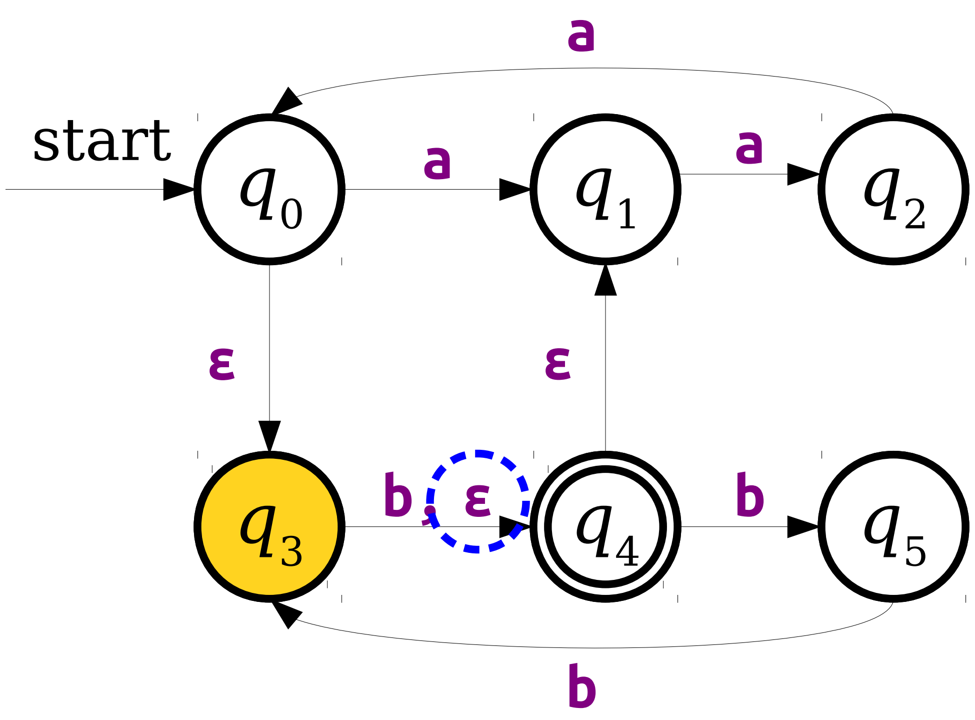

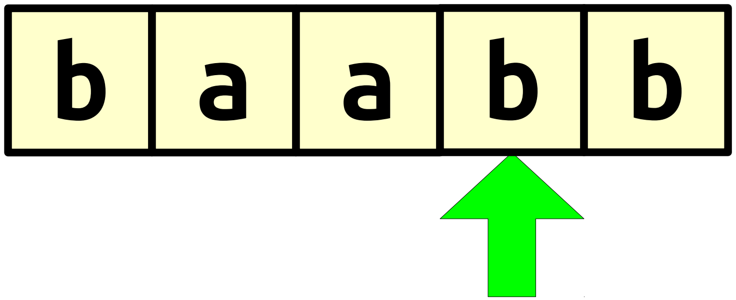

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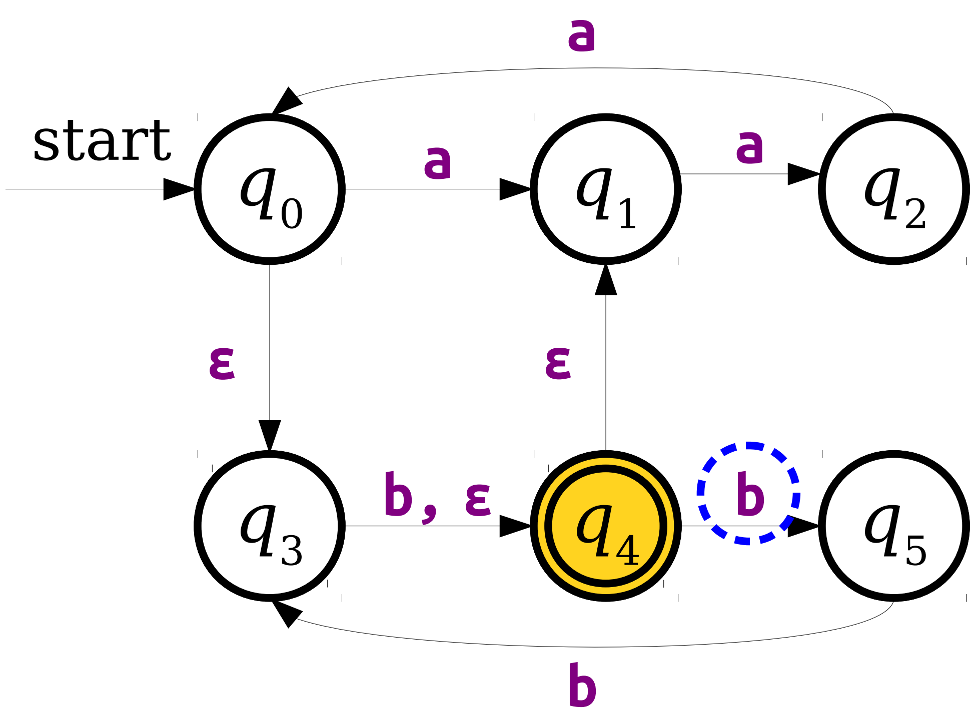

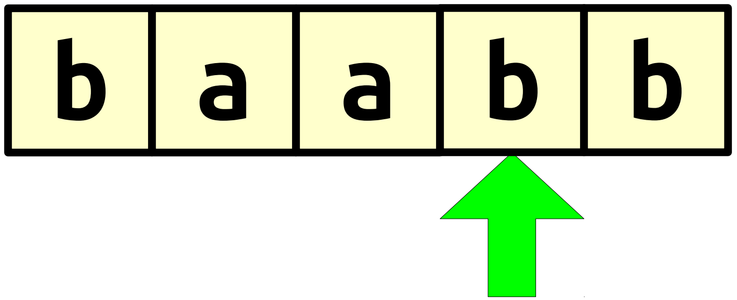

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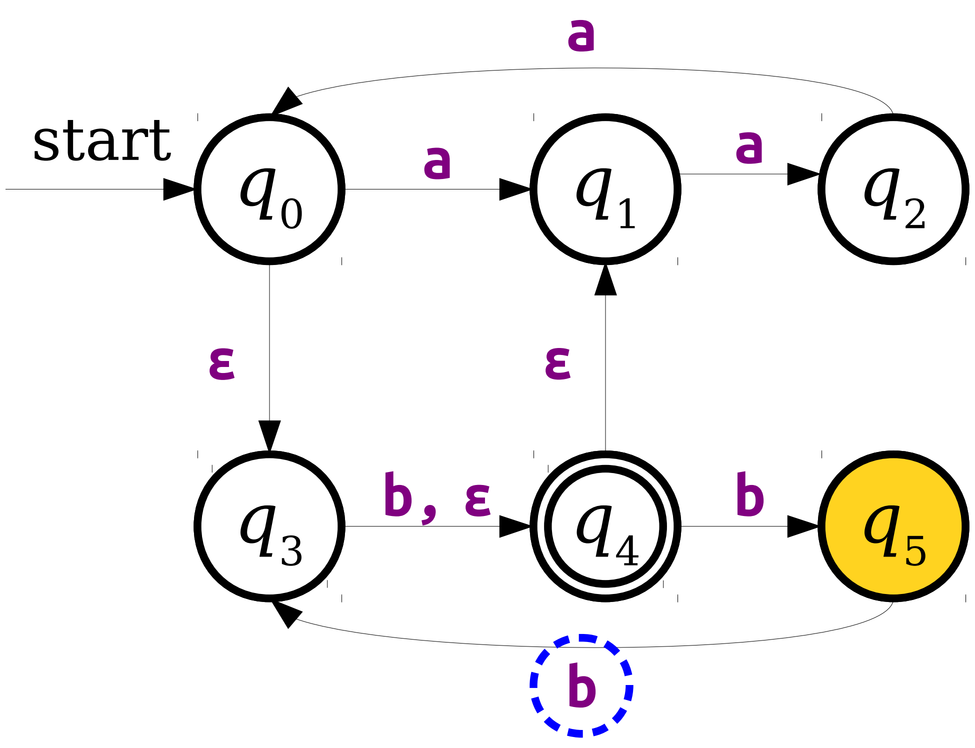

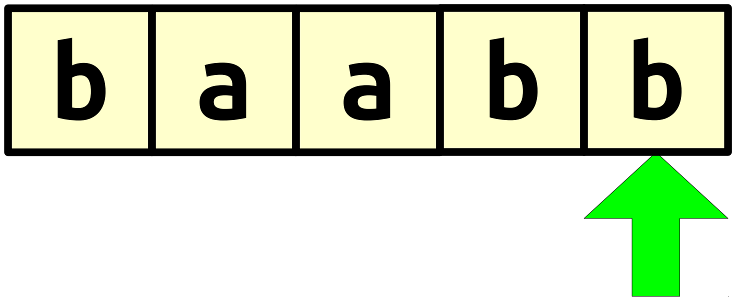

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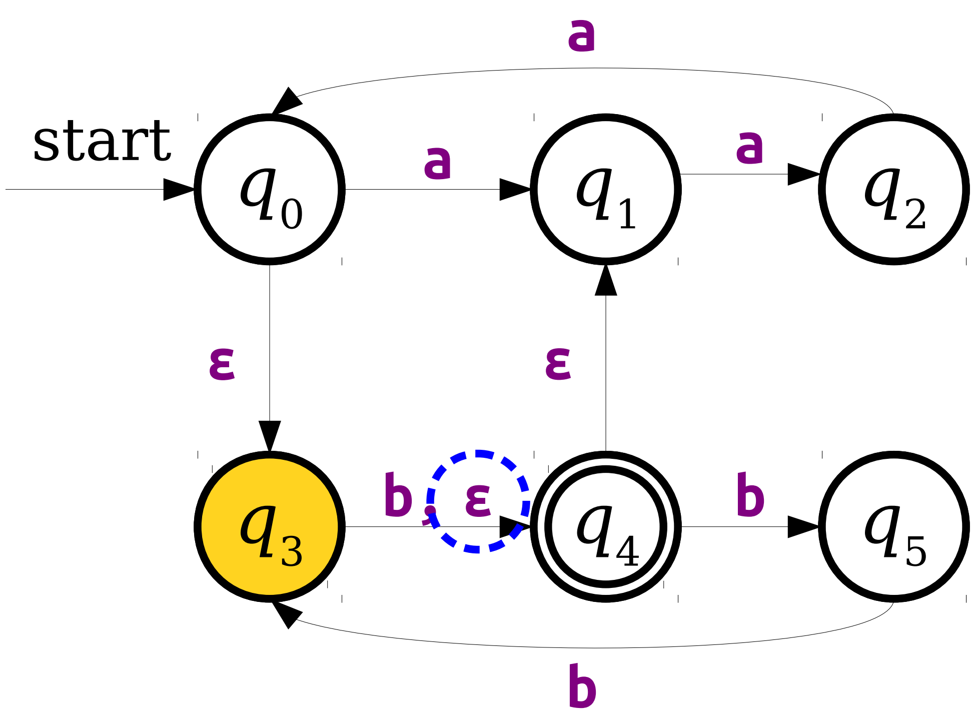

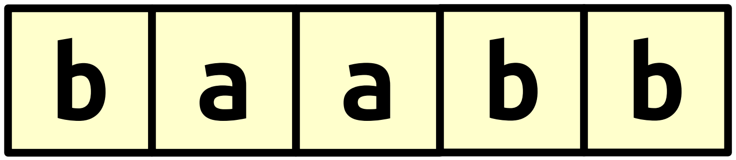

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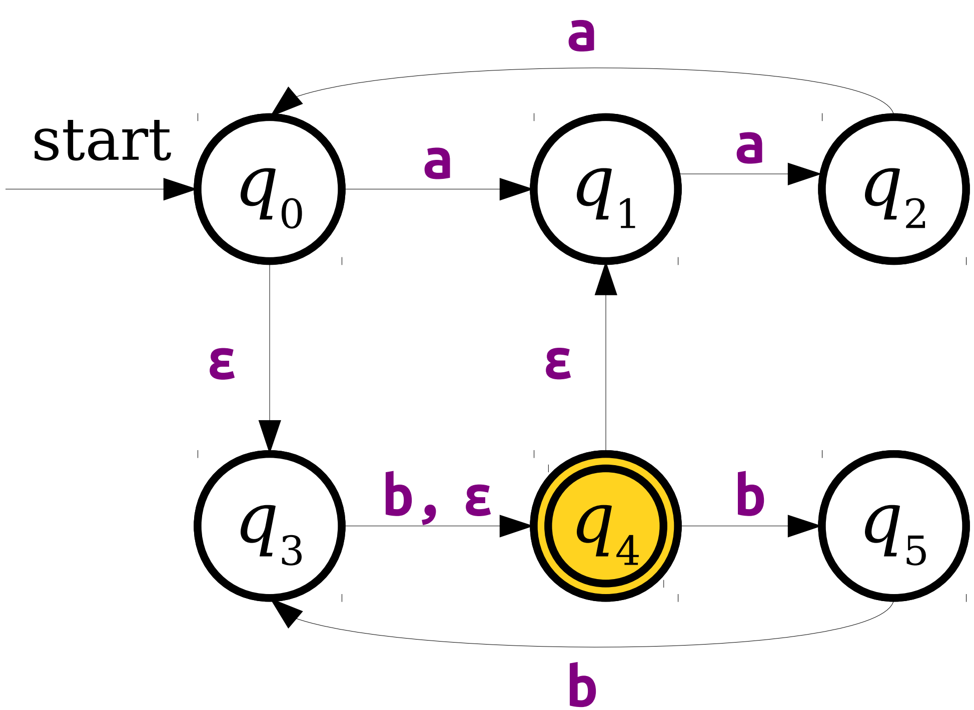

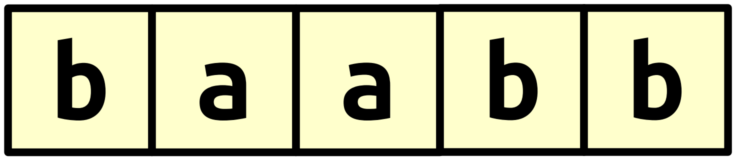

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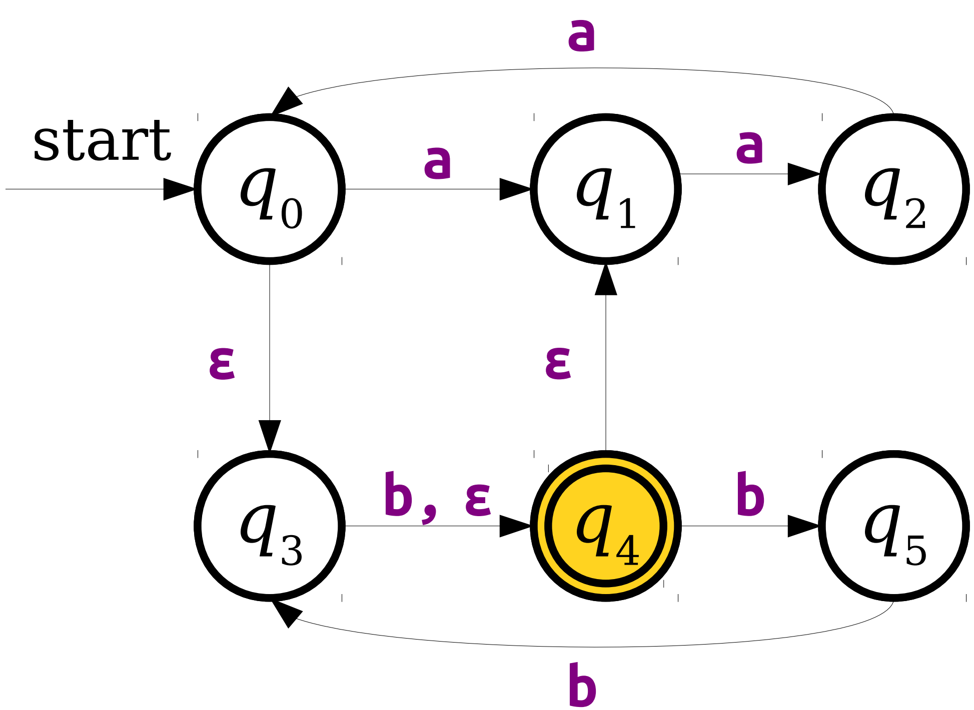
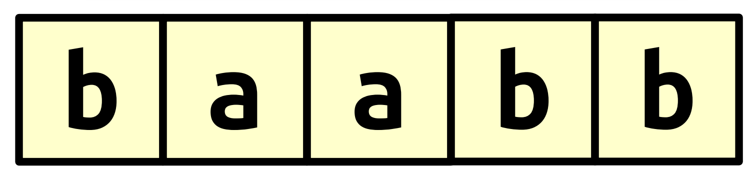




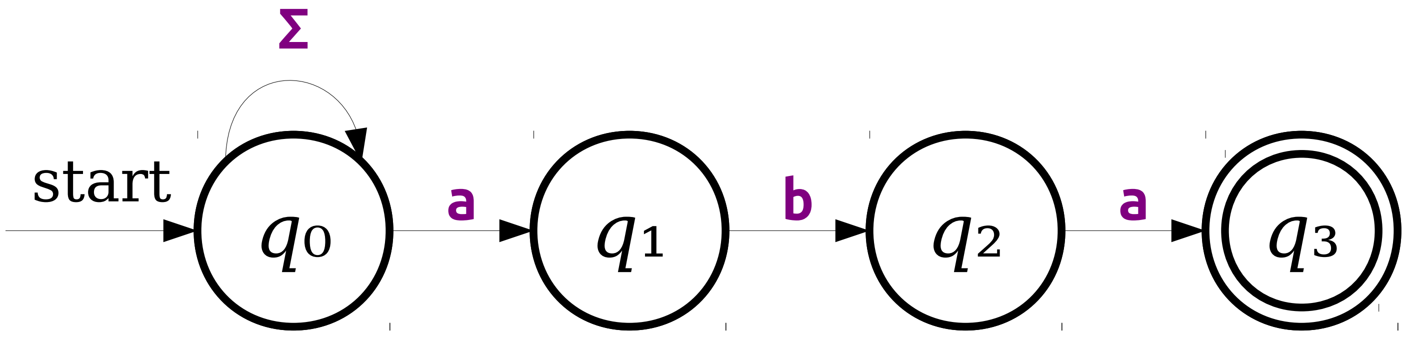
ε-Transitions

- NFAs have a special type of transition called the **ε-transition**.
- An NFA may follow any number of ε-transitions at any time without consuming any input.
- NFAs are not *required* to follow ε-transitions. It's simply another option at the machine's disposal.

Intuiting Nondeterminism

- Nondeterministic machines are a serious departure from physical computers. How can we build up an intuition for them?
- There are two particularly useful frameworks for interpreting nondeterminism:
  - ***Perfect positive guessing***
  - ***Massive parallelism***

Perfect Positive Guessing



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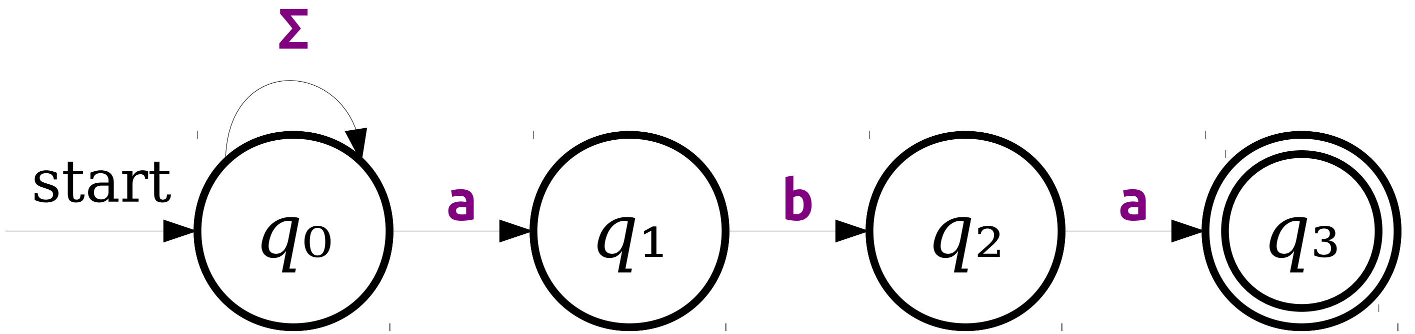

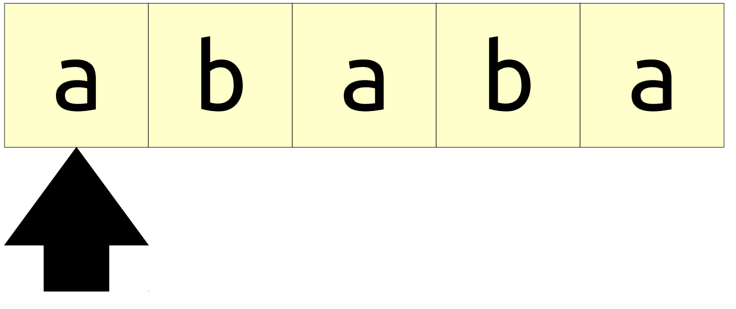

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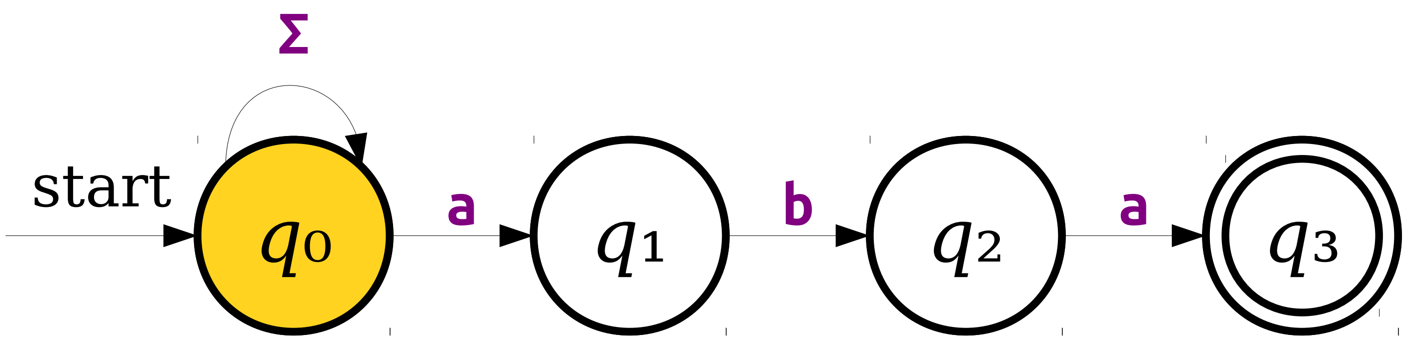

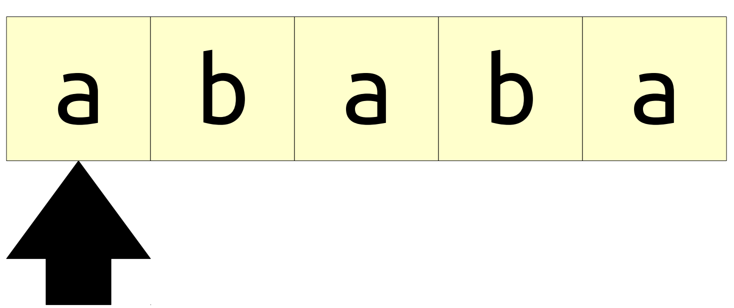

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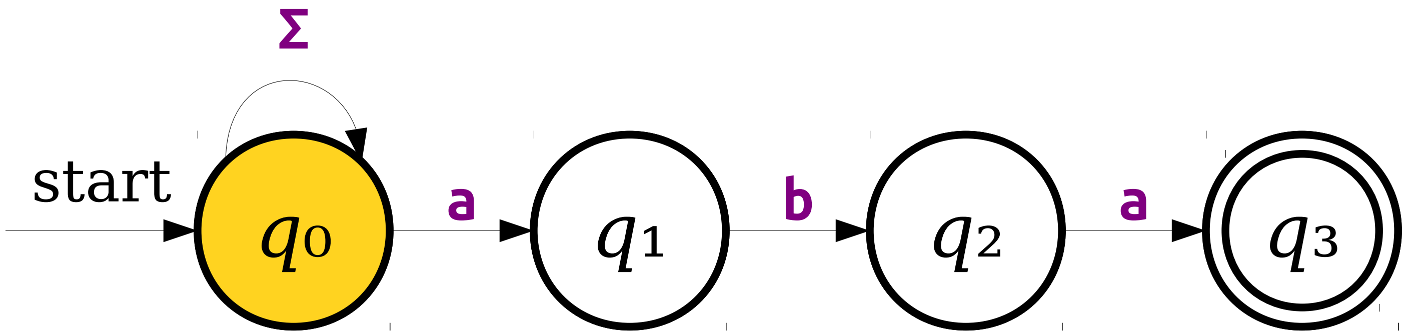

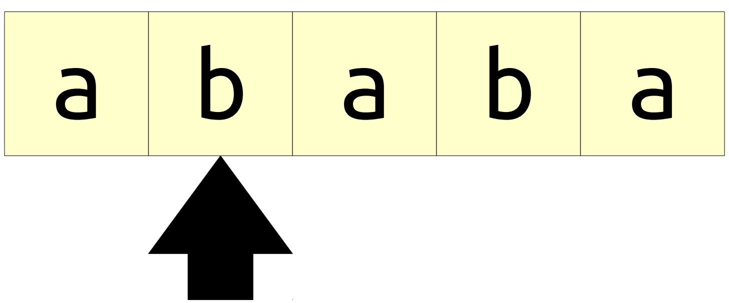

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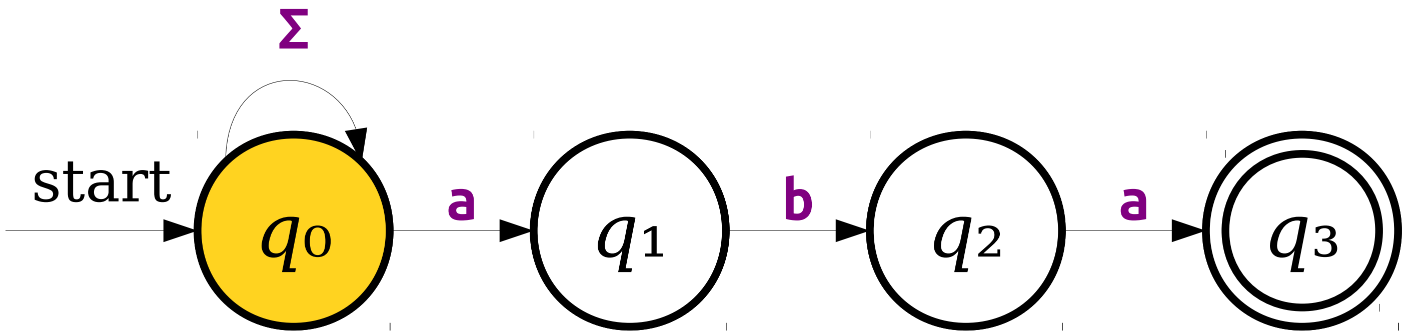

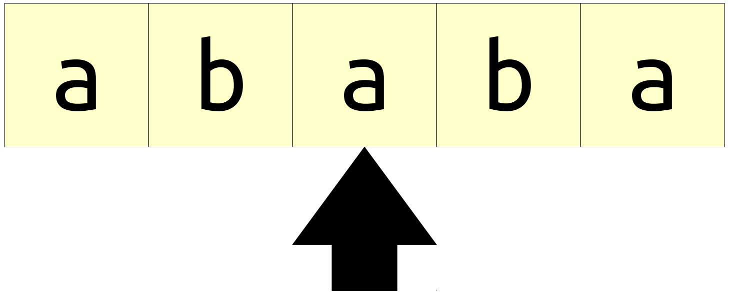

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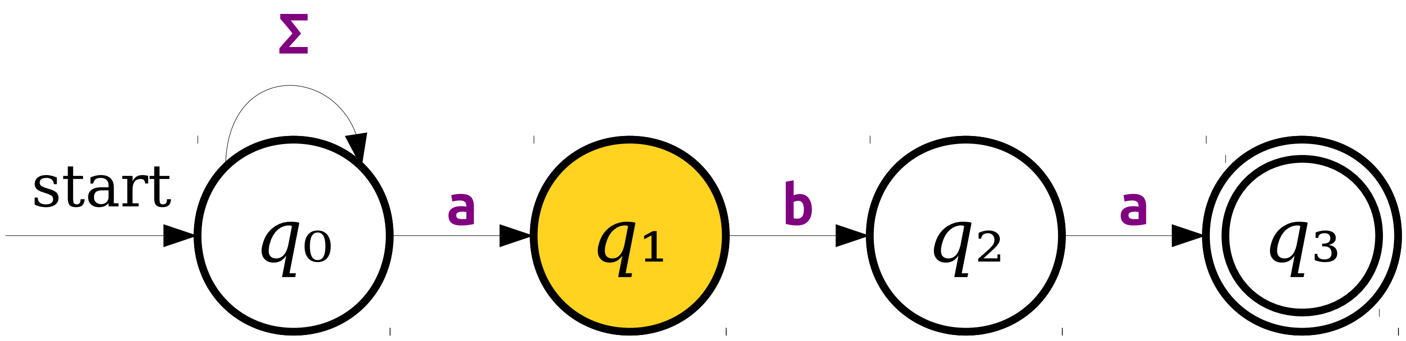

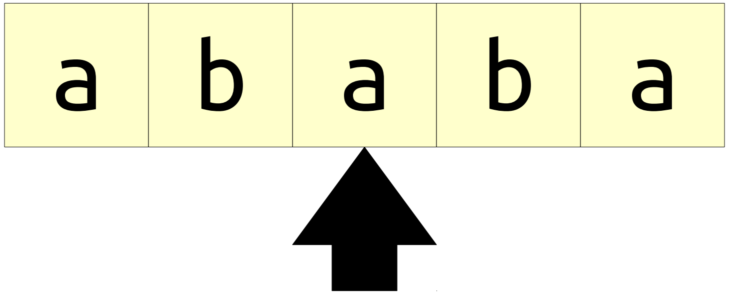

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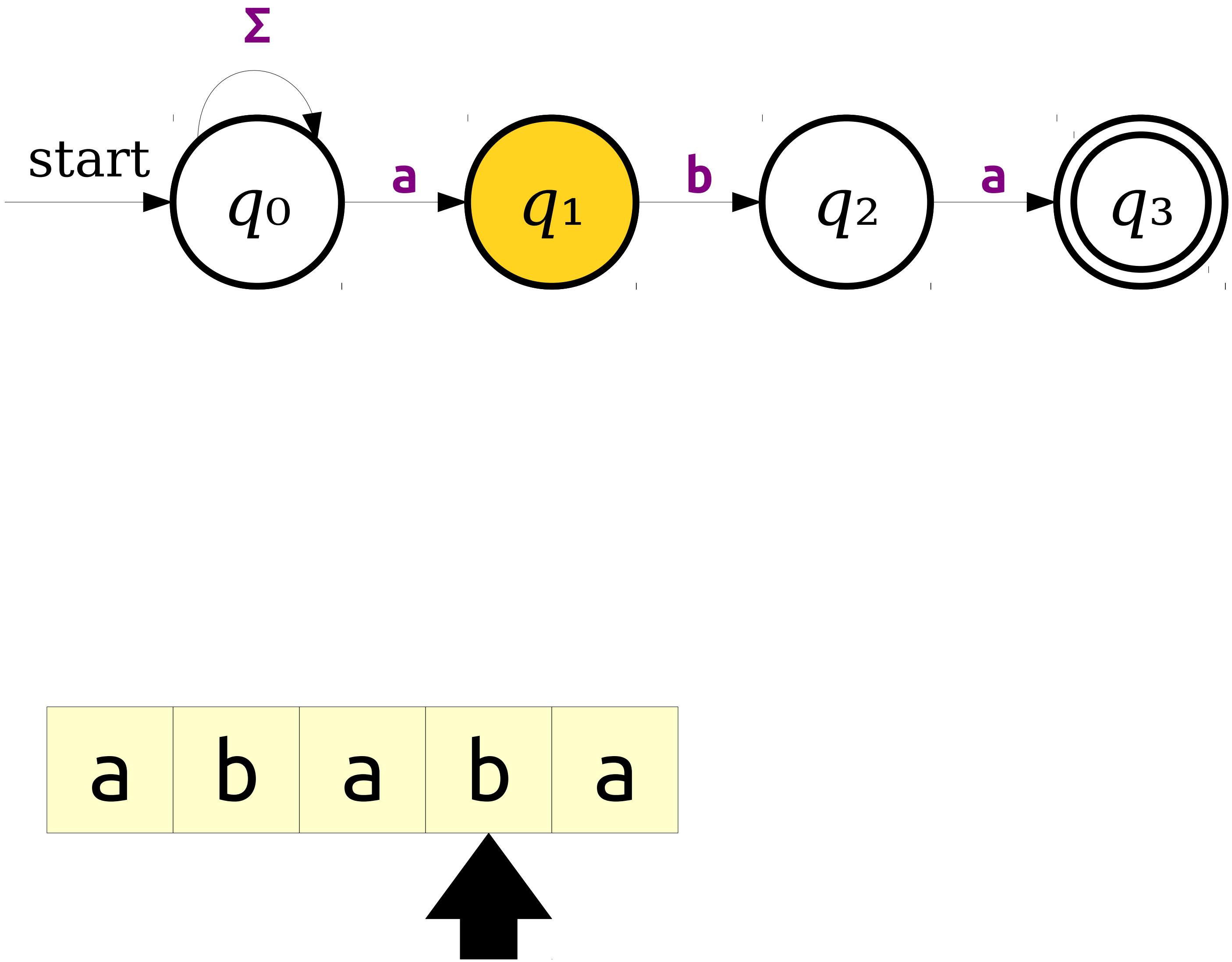

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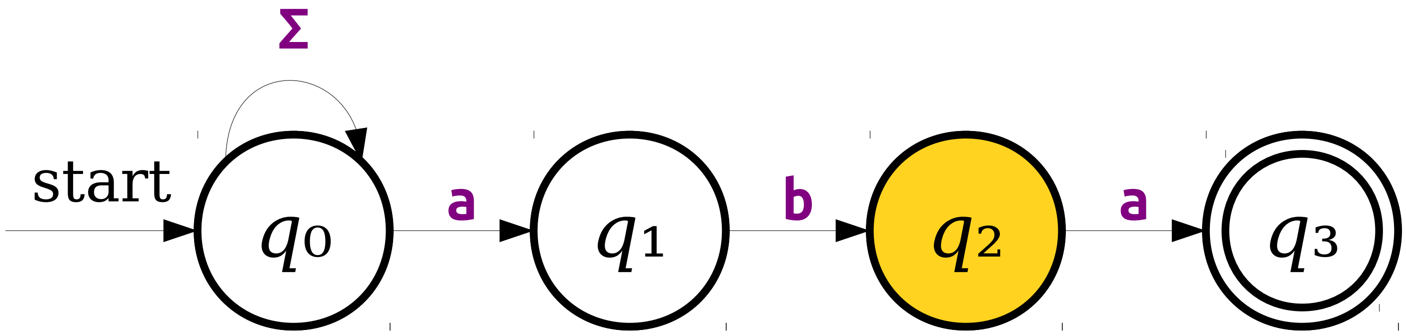

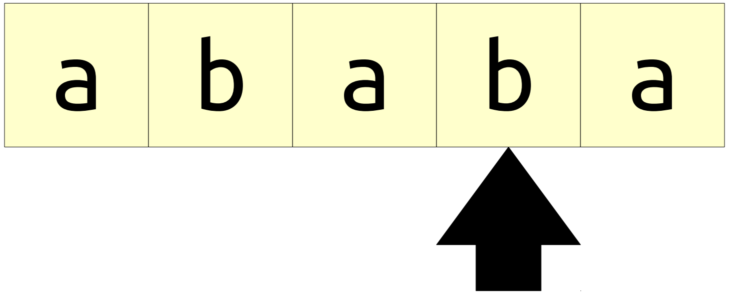

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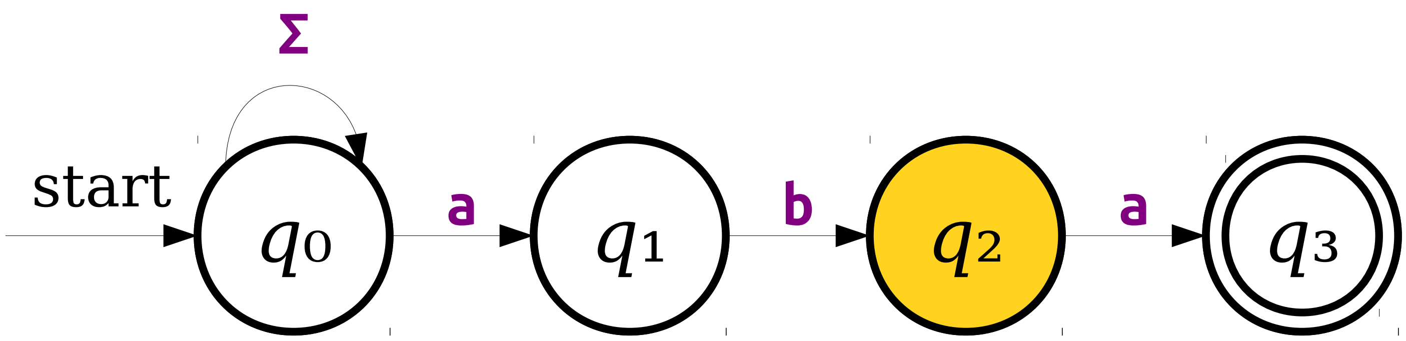

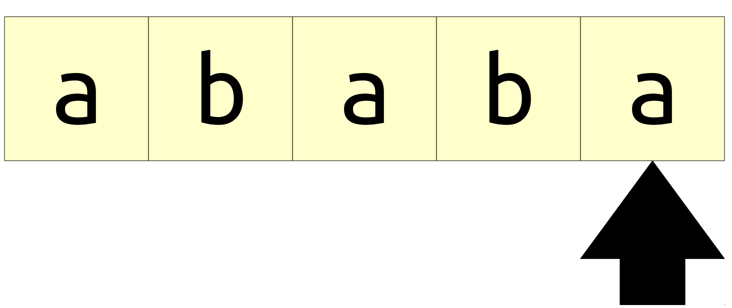

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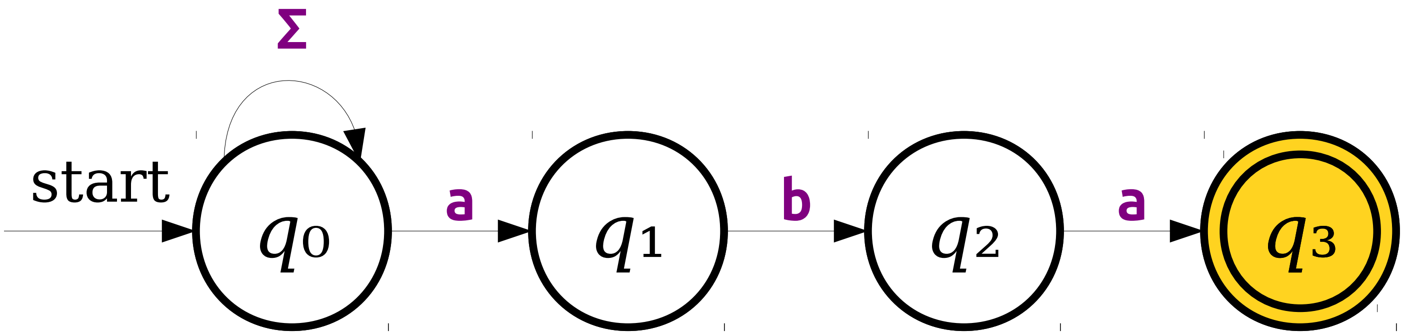

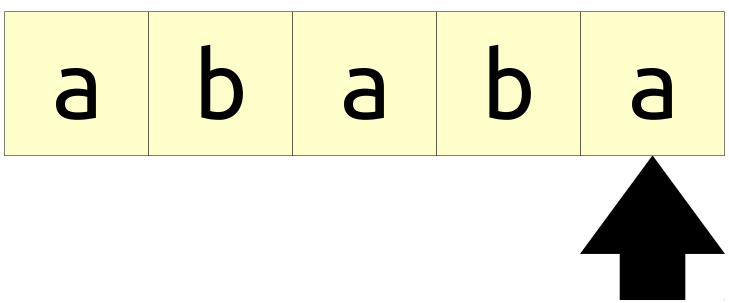

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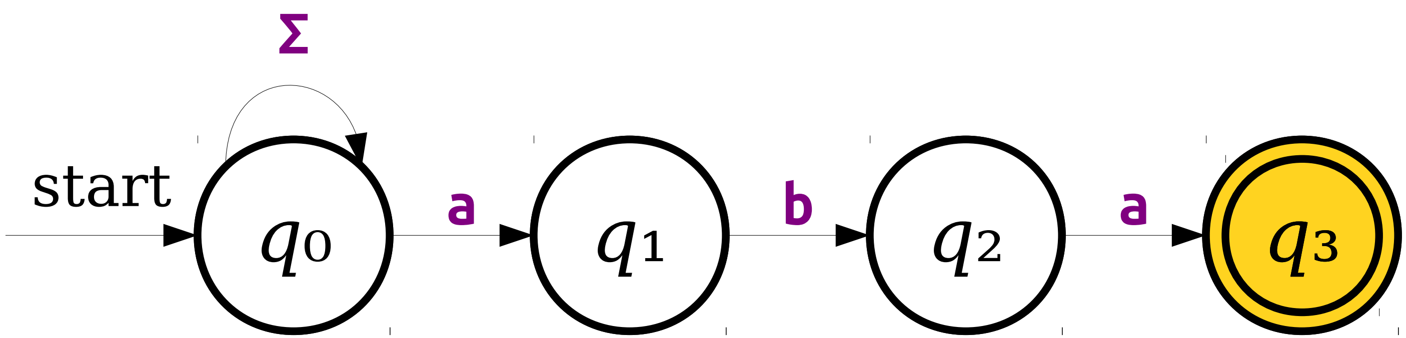
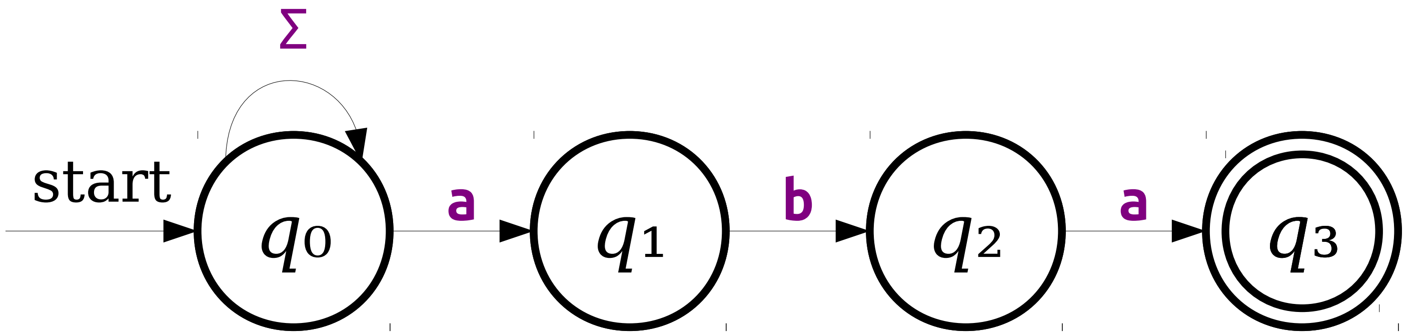


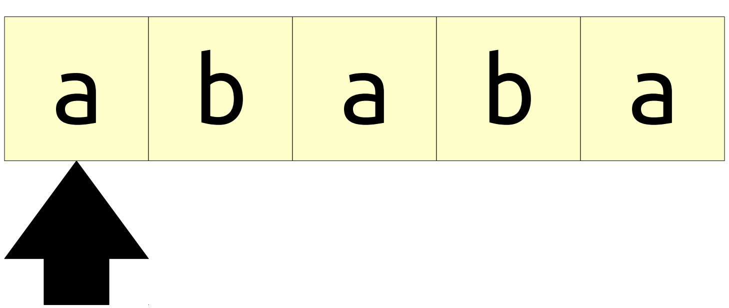
a	b	a	b	a

Perfect Positive Guessing

- We can view nondeterministic machines as having ***Magic Superpowers*** that enable them to guess choices that lead to an accepting state.
  - If there is at least one choice that leads to an accepting state, the machine will guess it.
  - If there are no choices, the machine guesses any one of the wrong guesses.
- There is no known way to physically model this intuition of nondeterminism – this is quite a departure from reality!

Massive Parallelism





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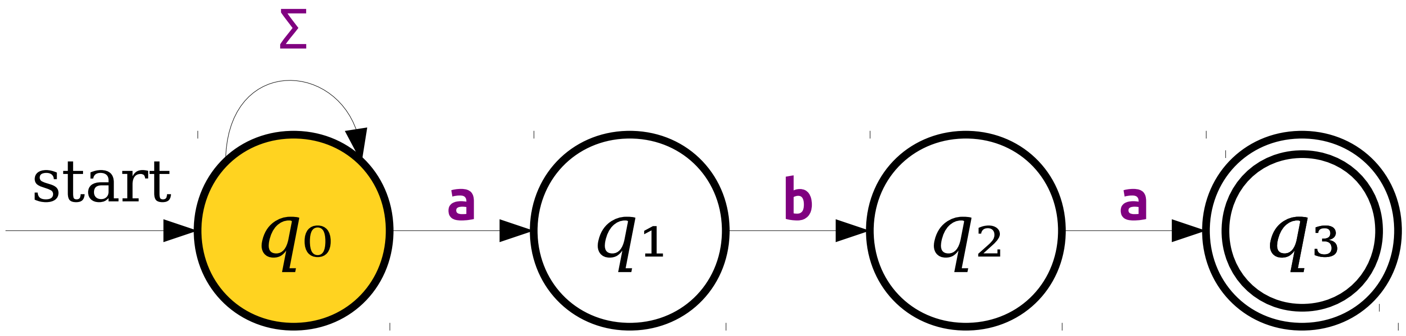

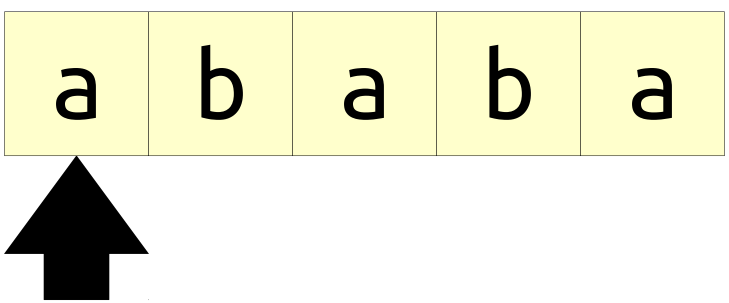

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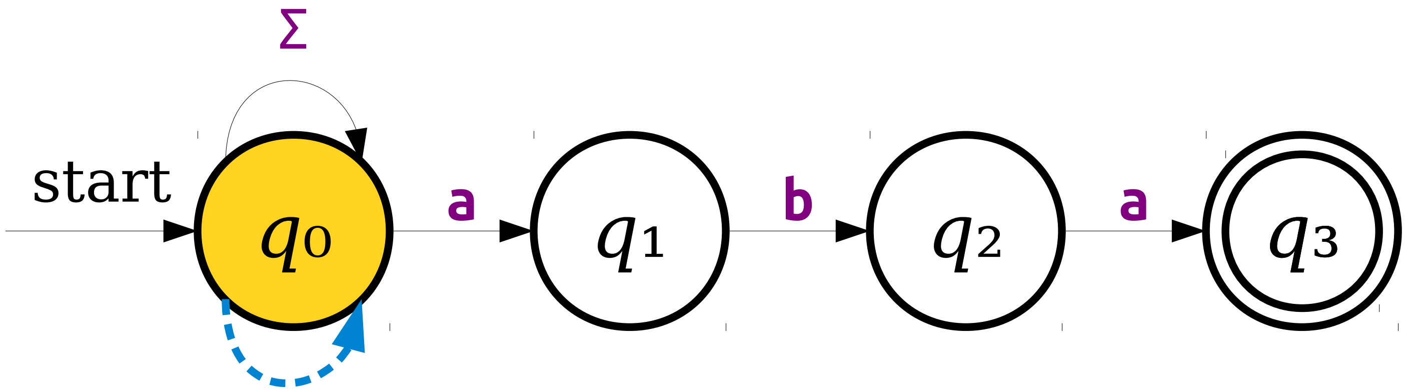

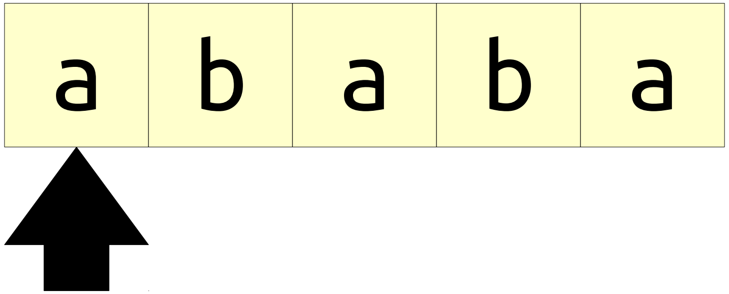

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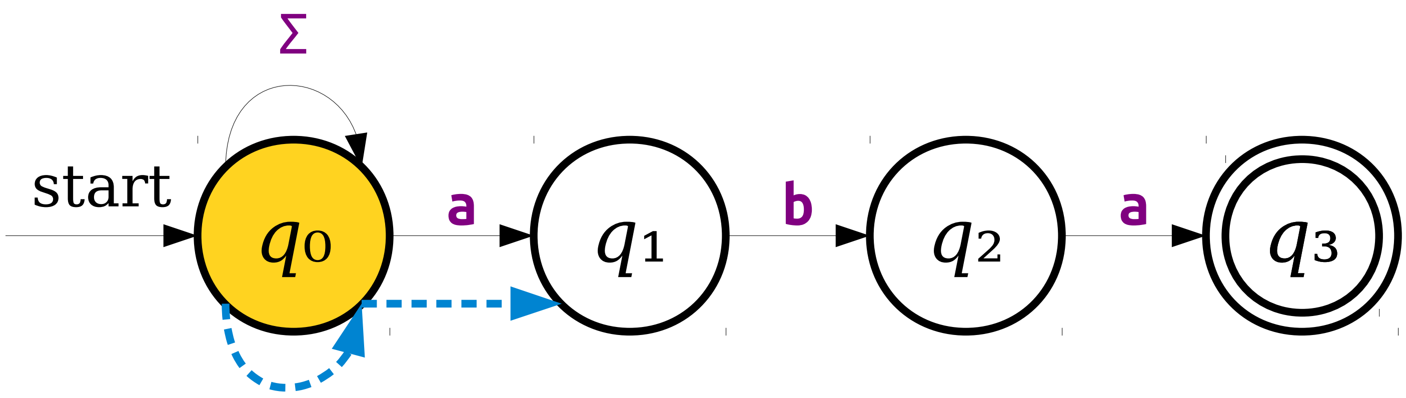

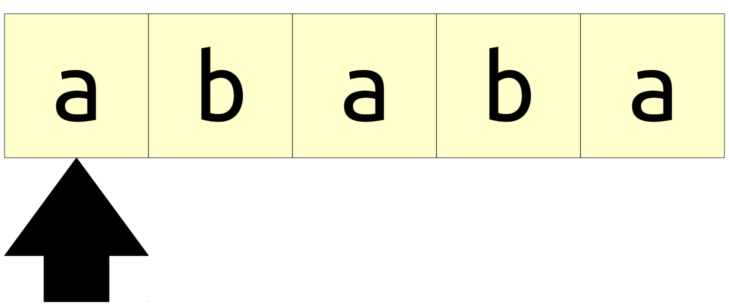

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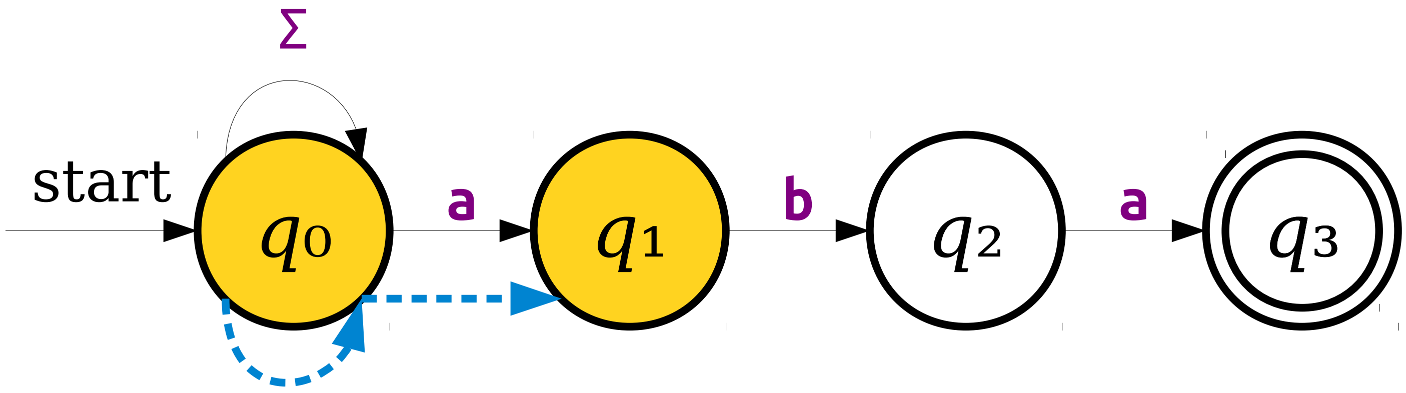

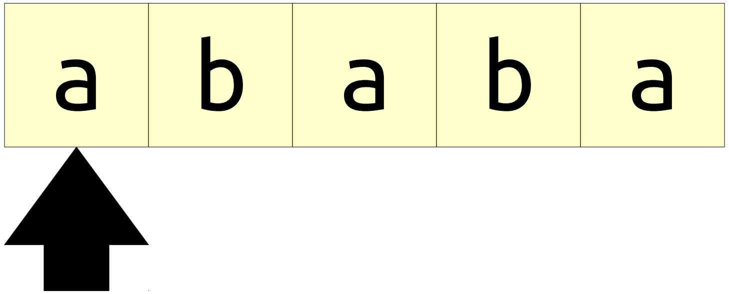

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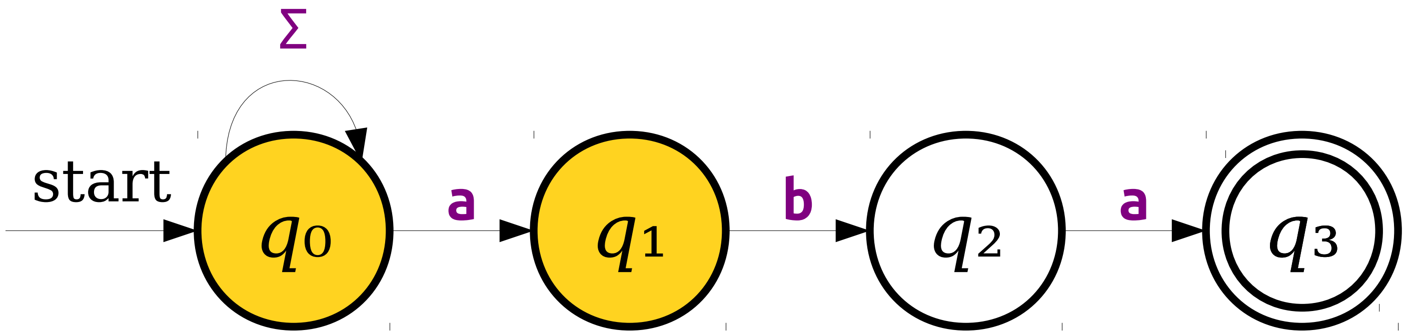

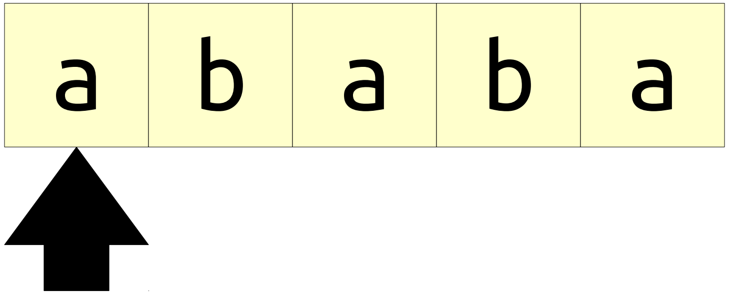

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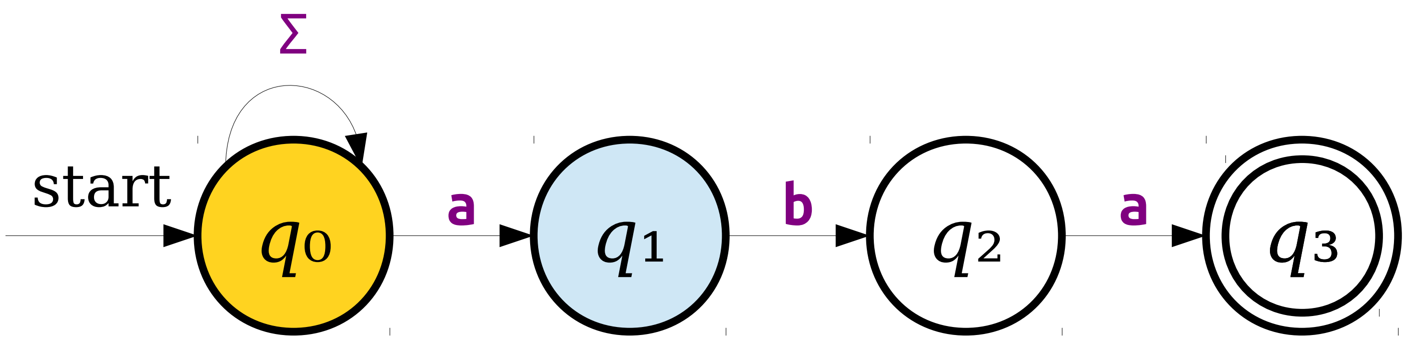

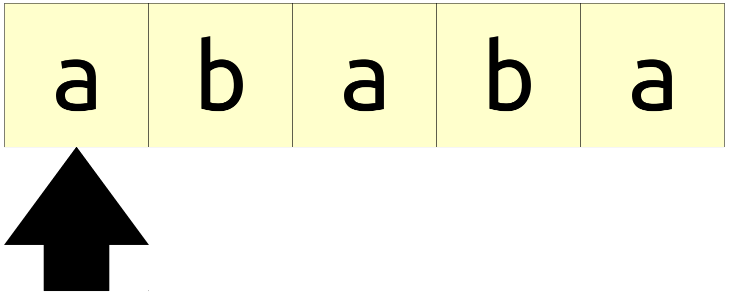

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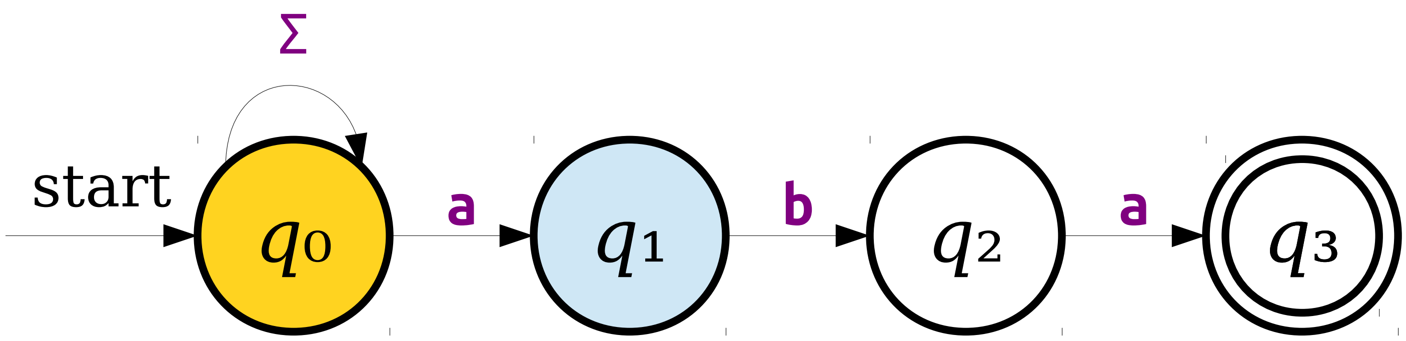

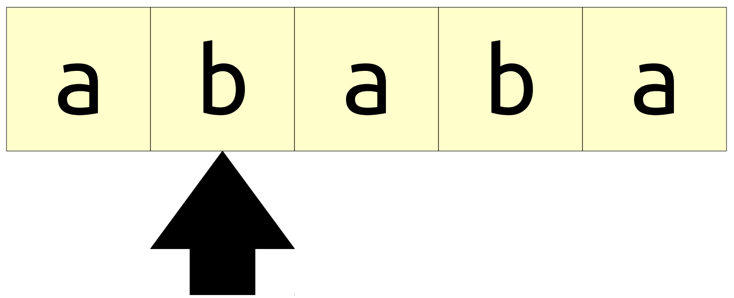

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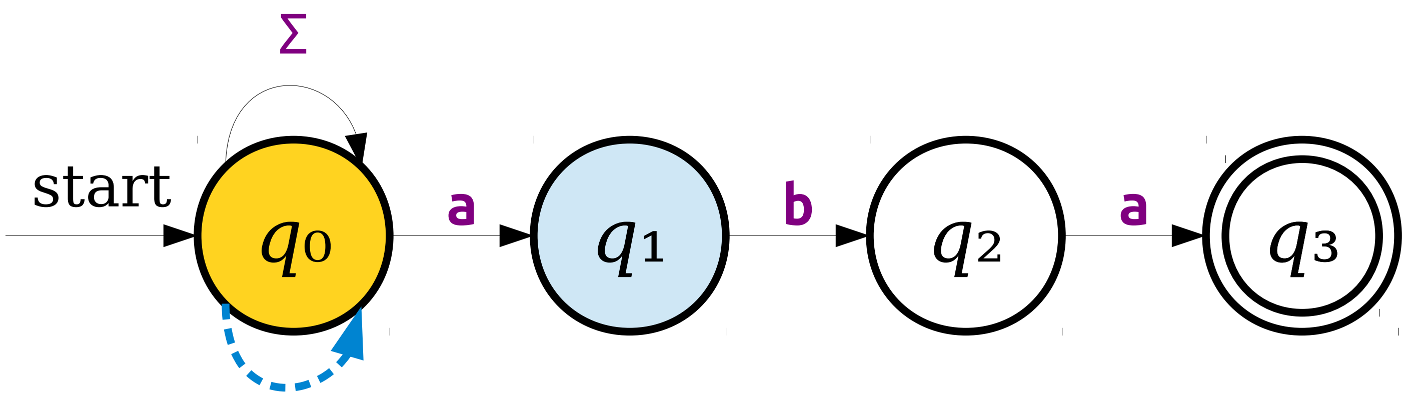

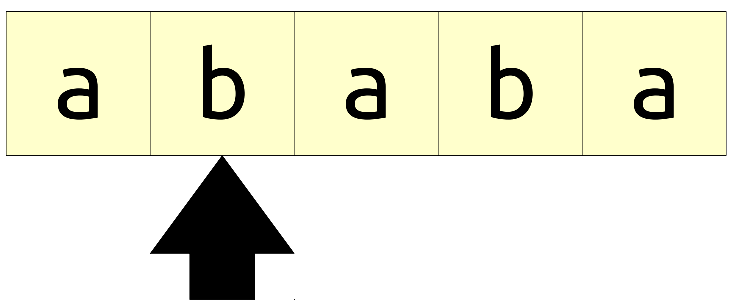

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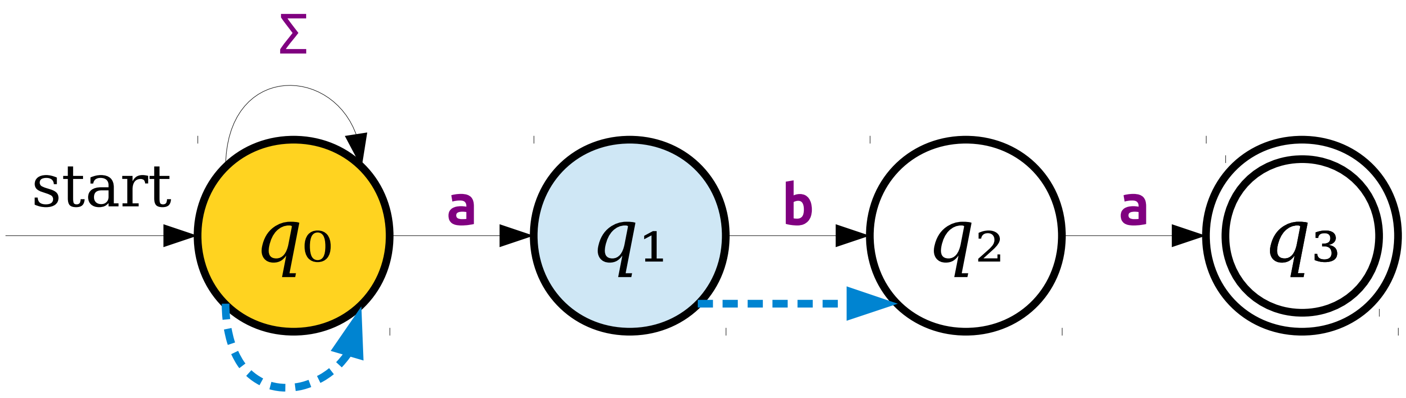

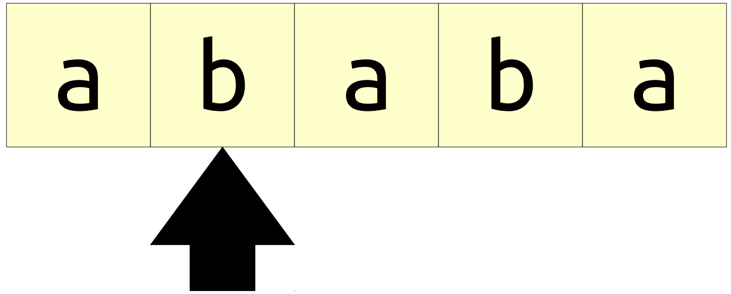

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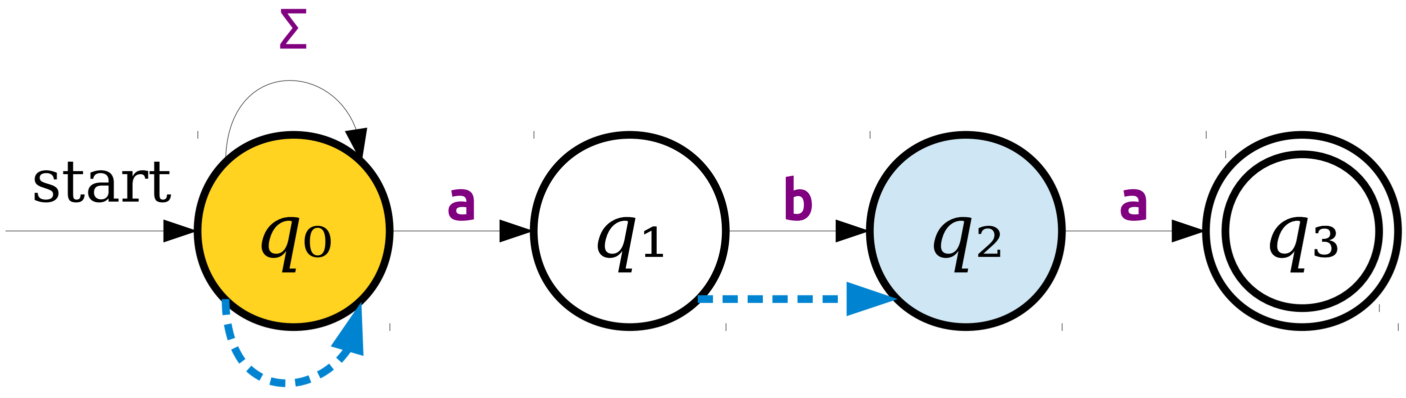

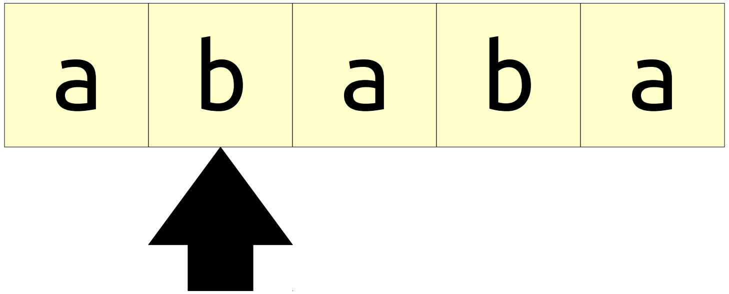

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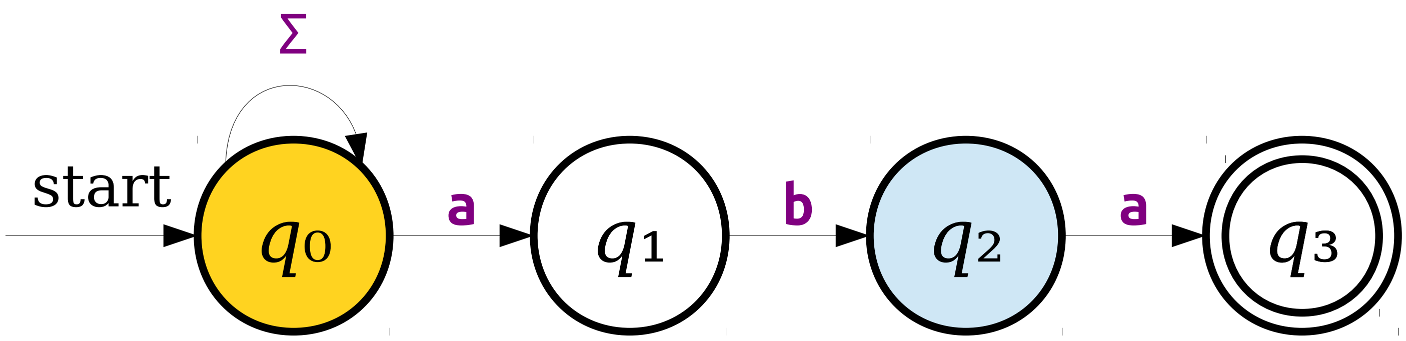

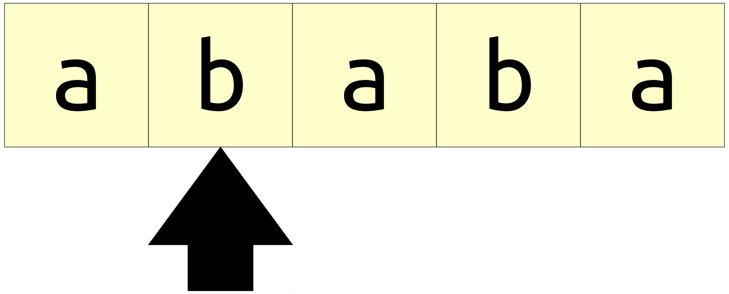

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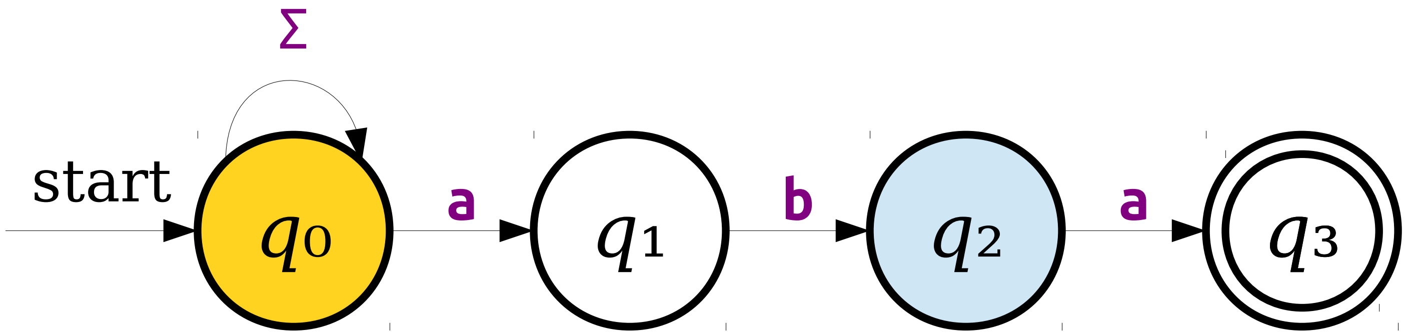

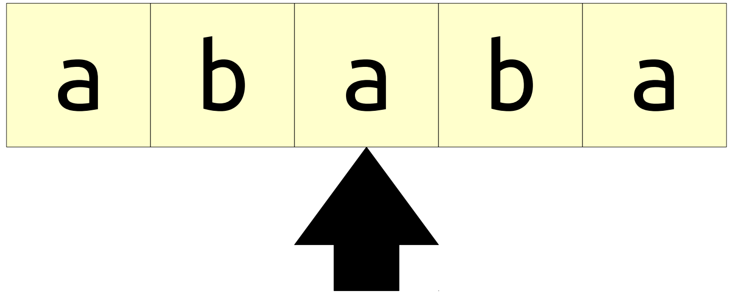

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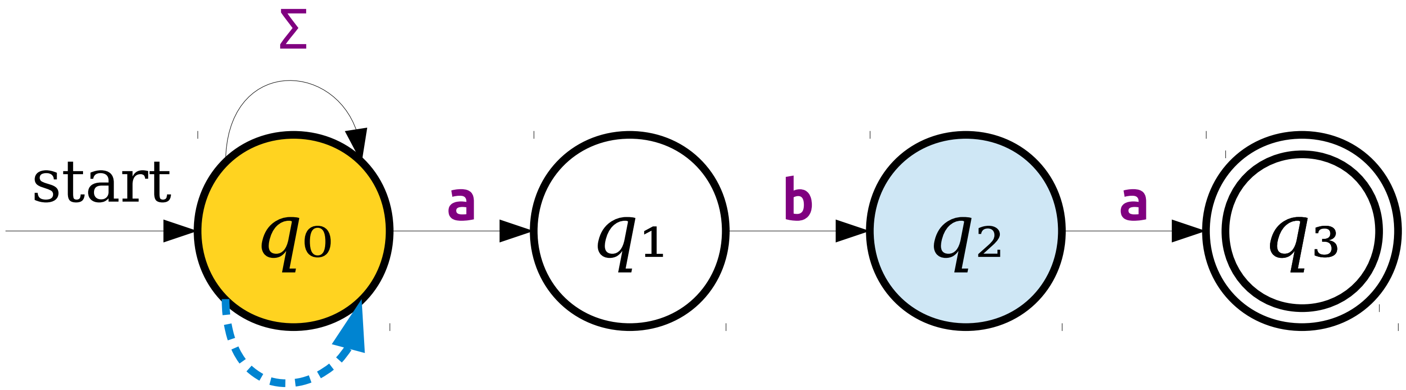

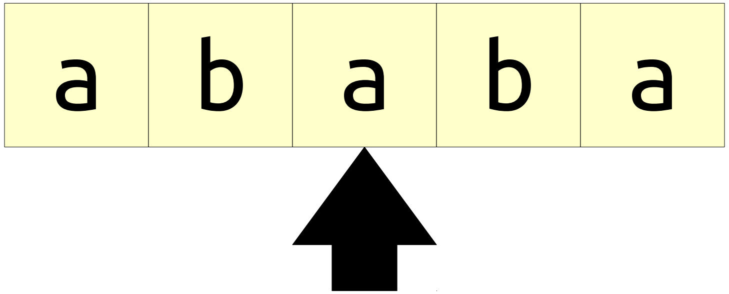

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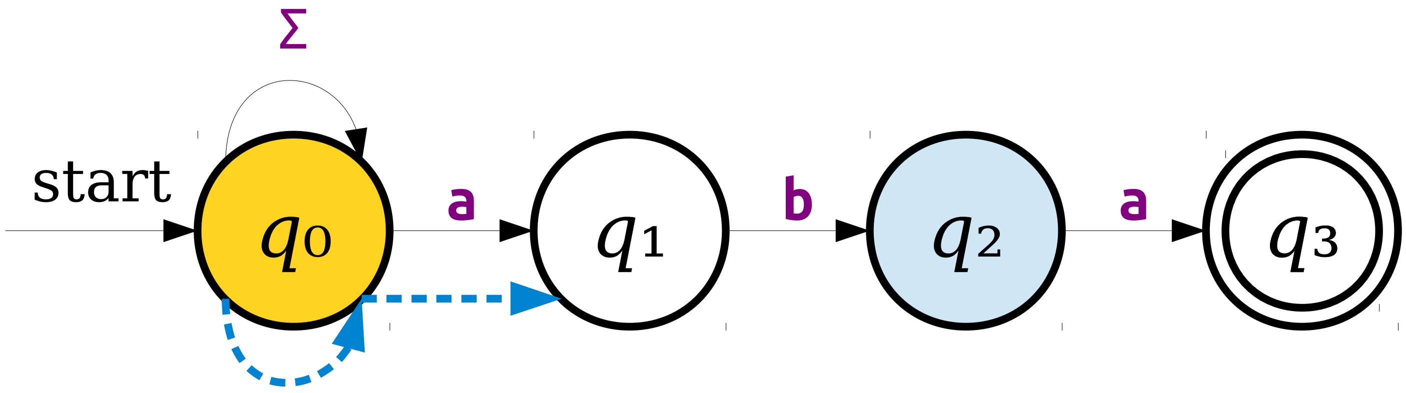

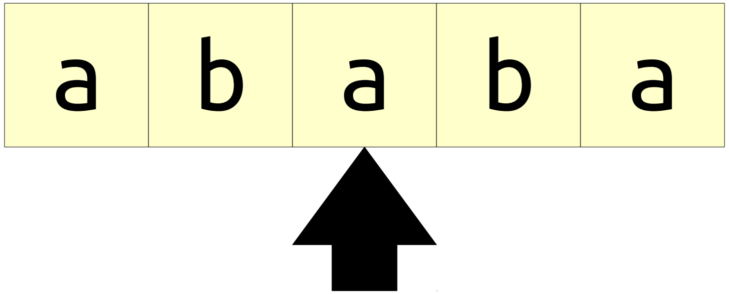

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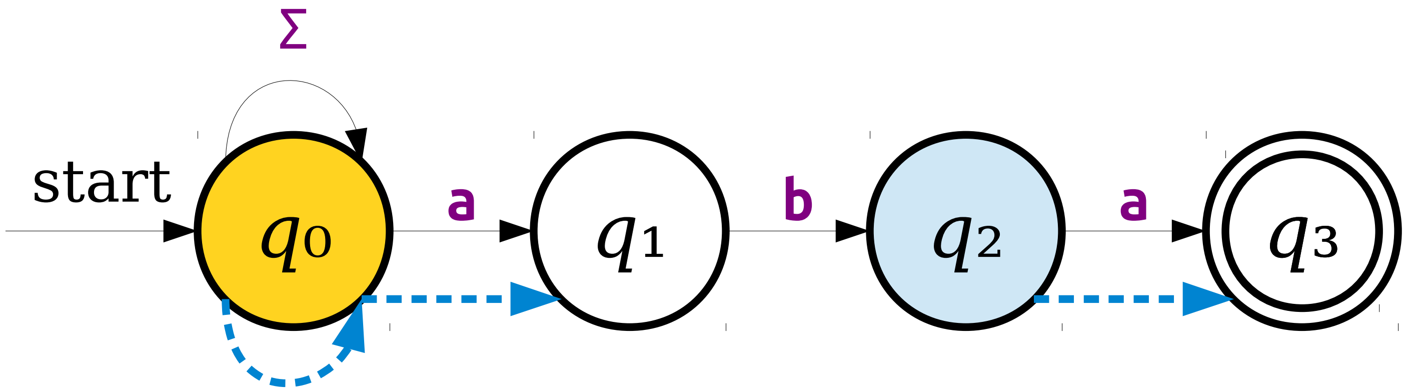

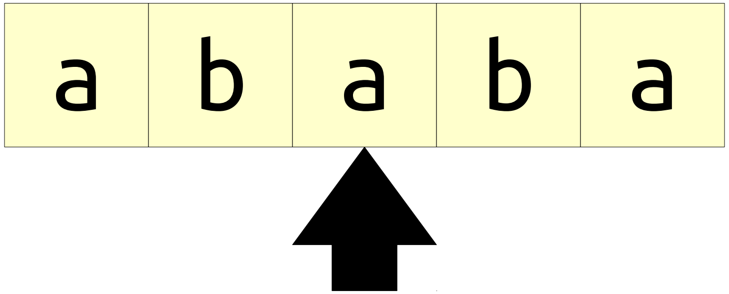

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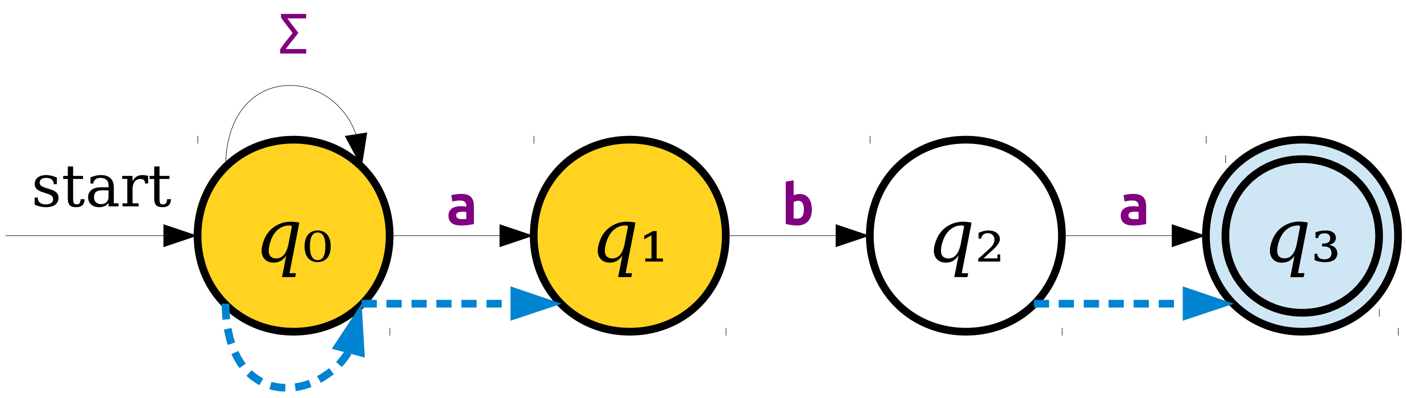

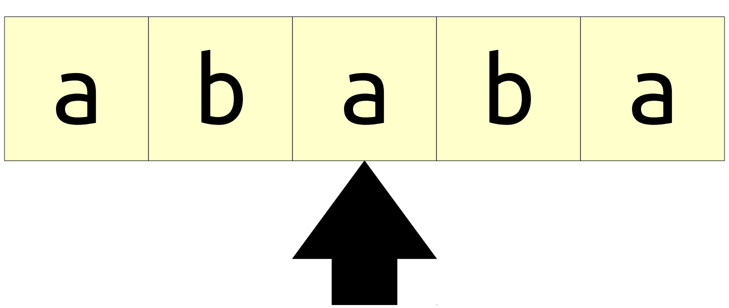

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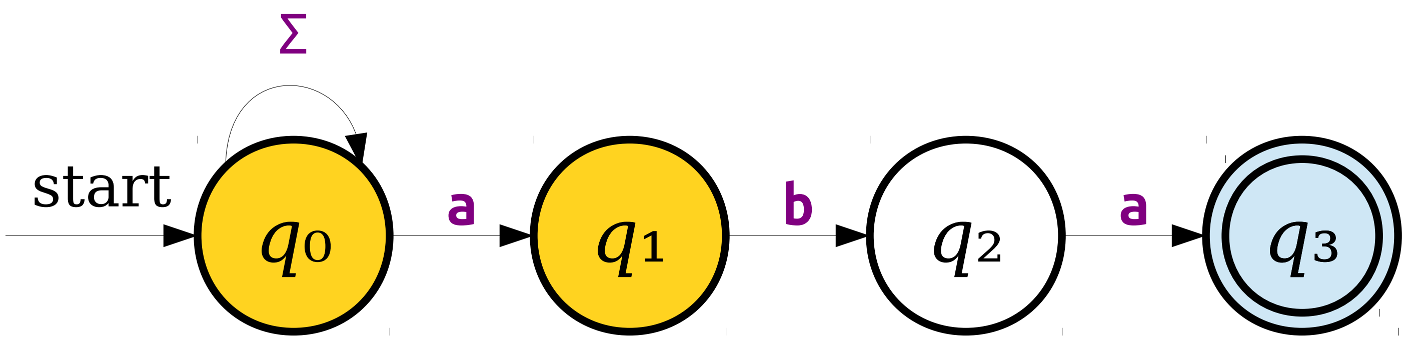
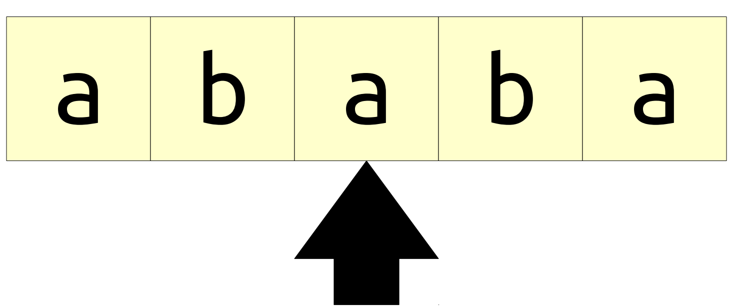
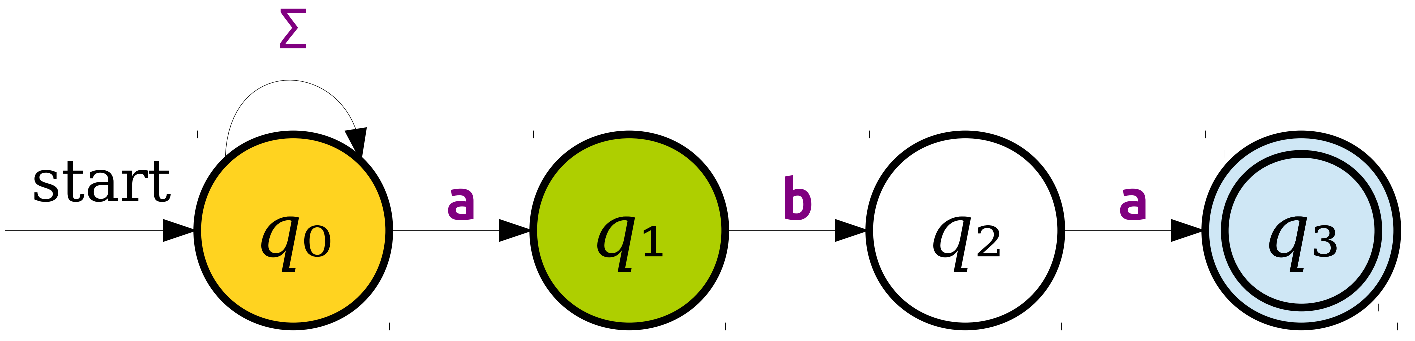
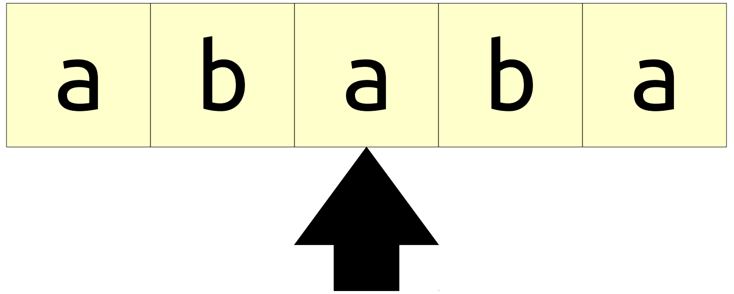




Massive Parallelism





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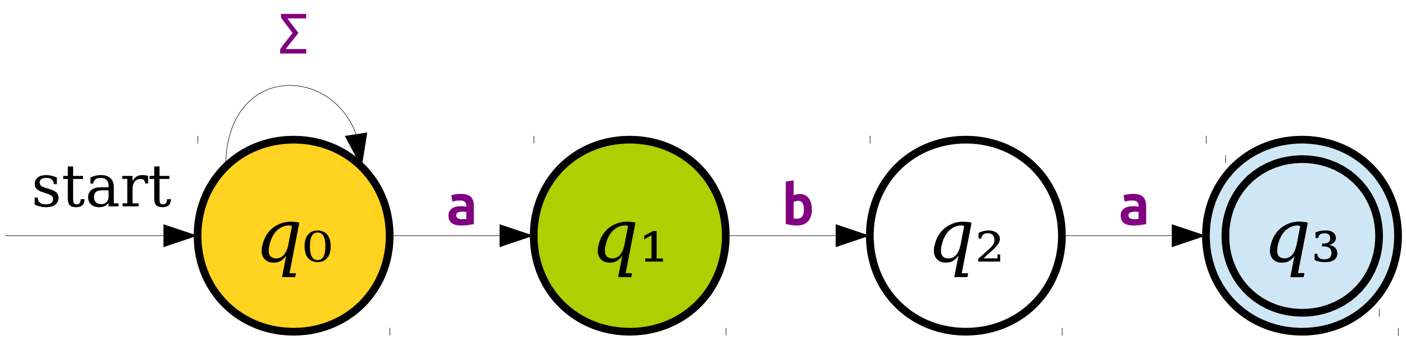

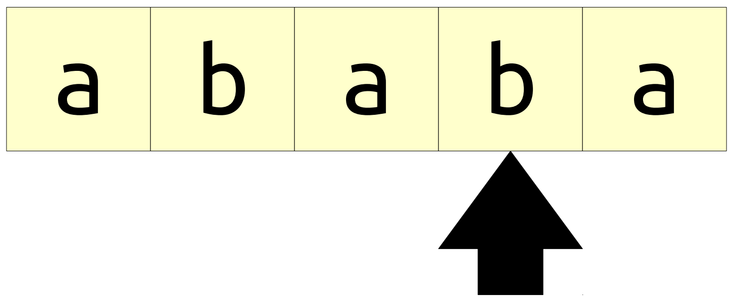

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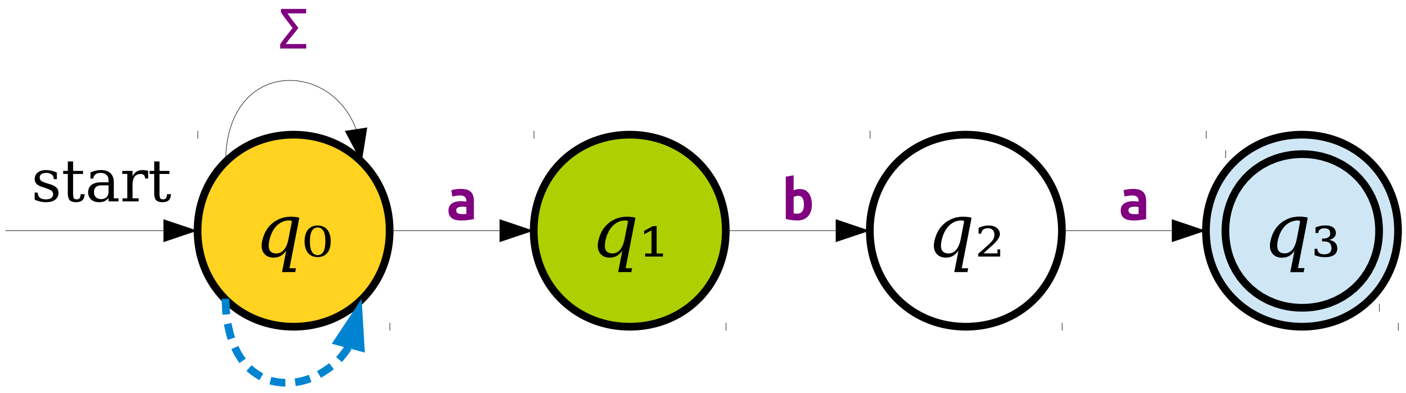

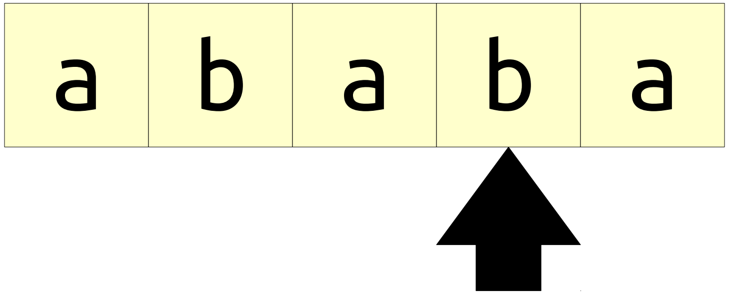

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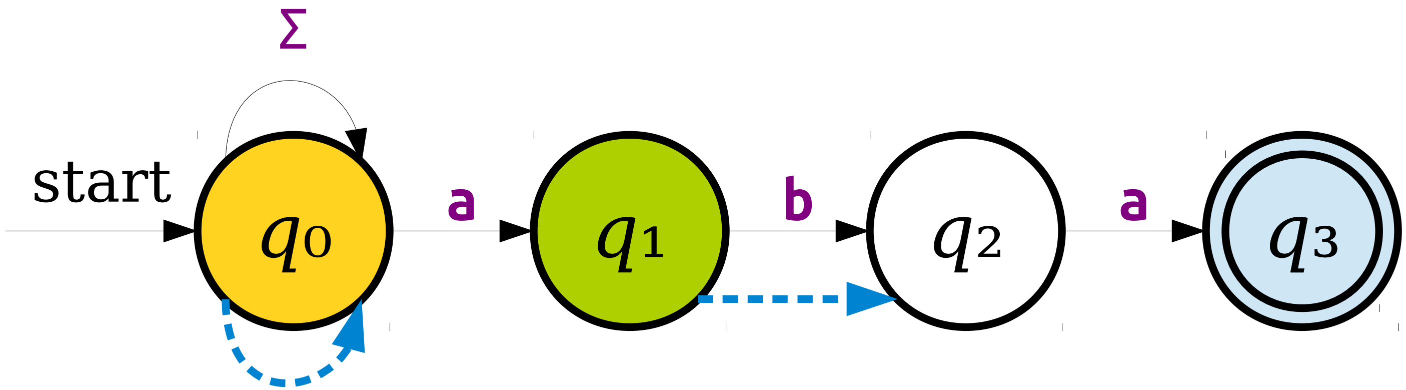

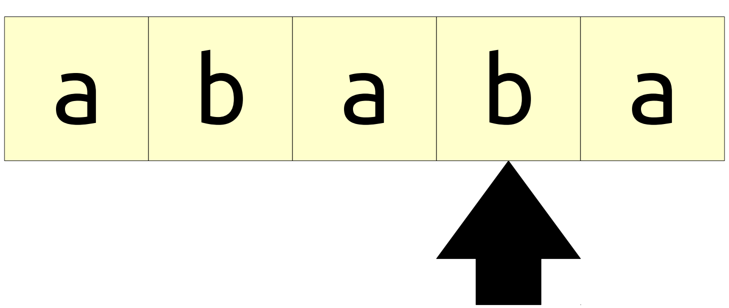

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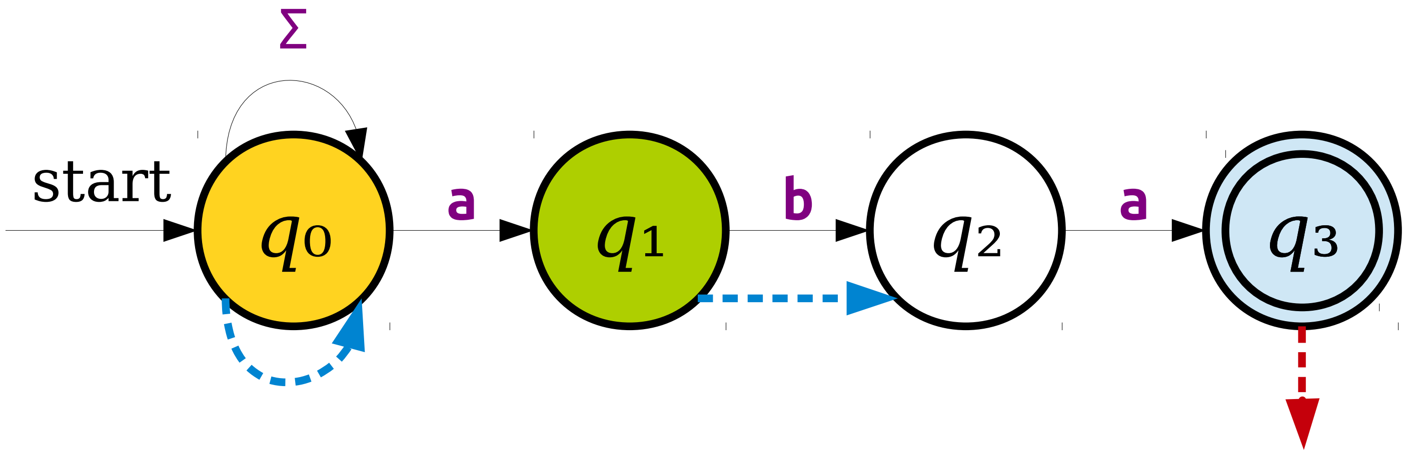

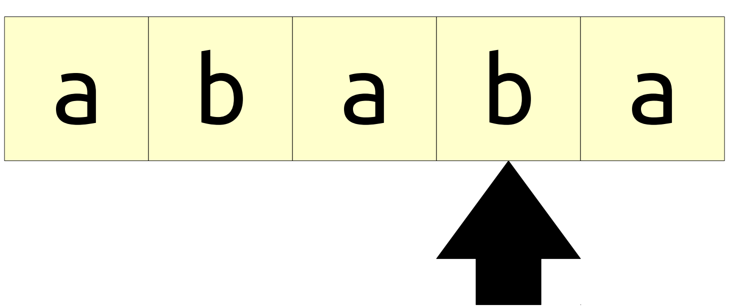

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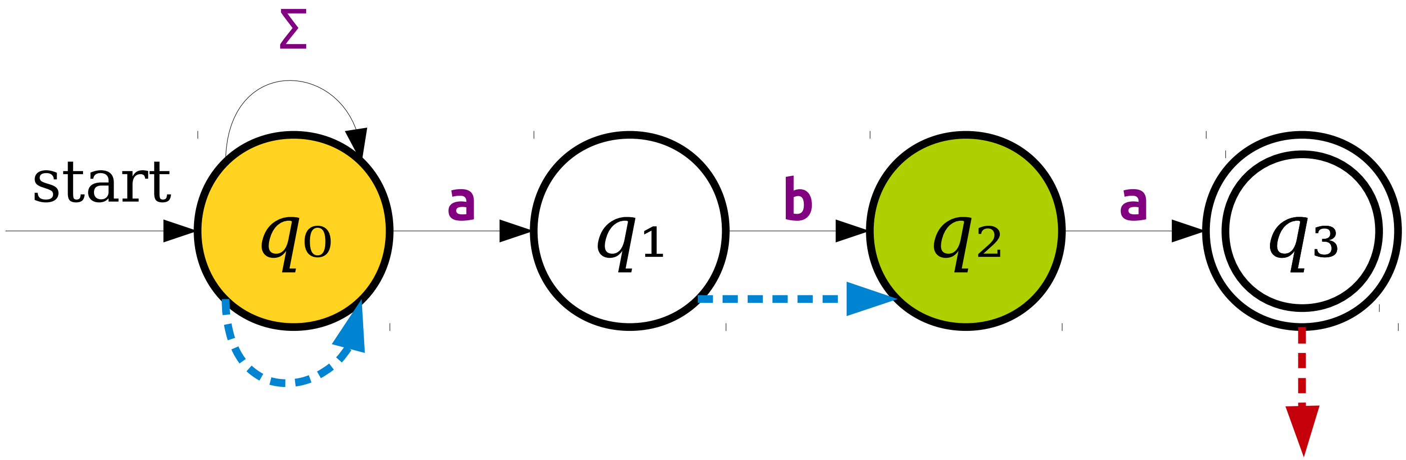

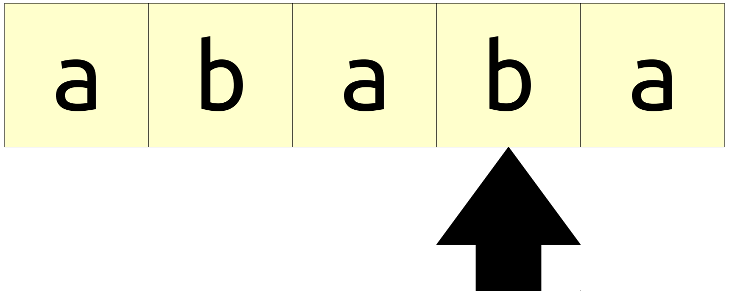

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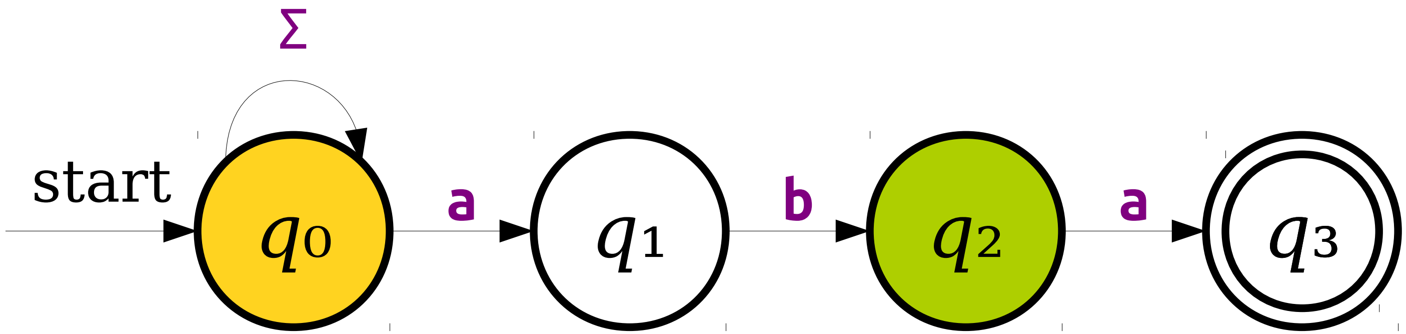

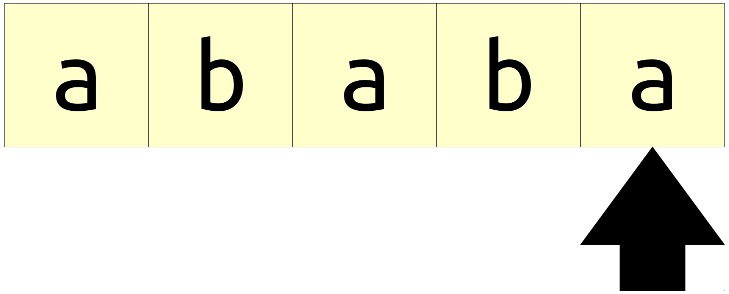

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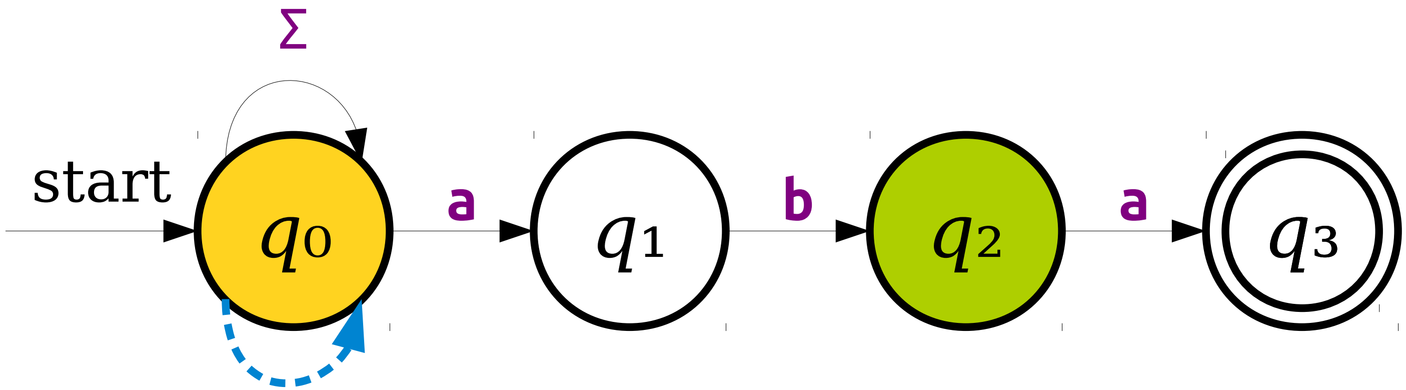

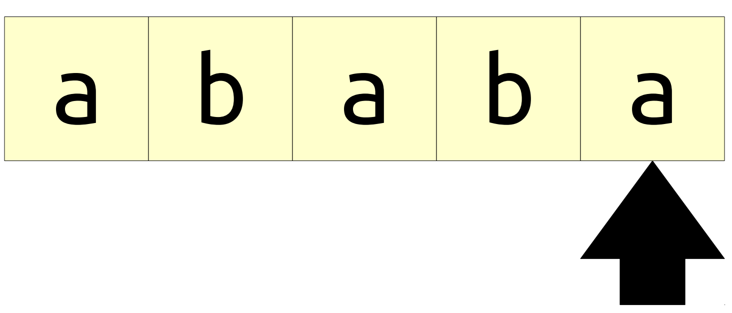

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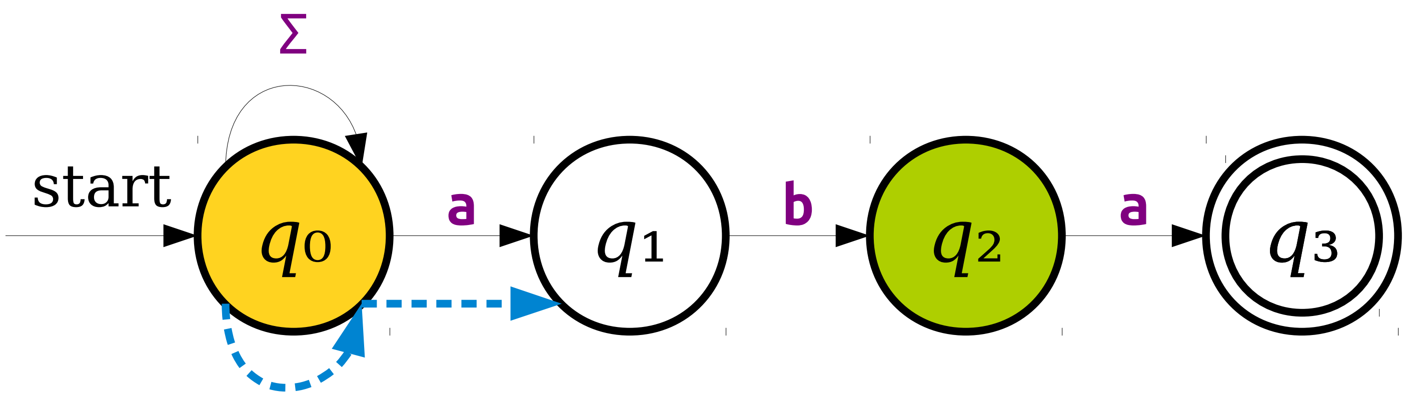

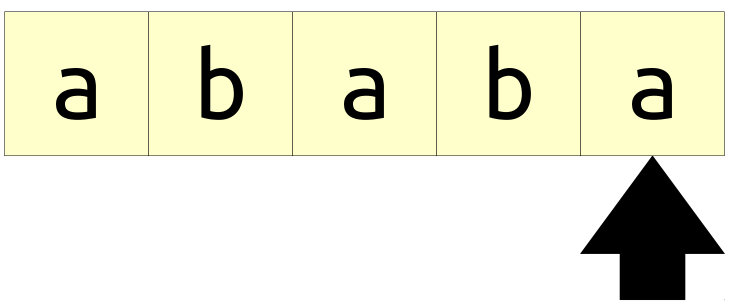

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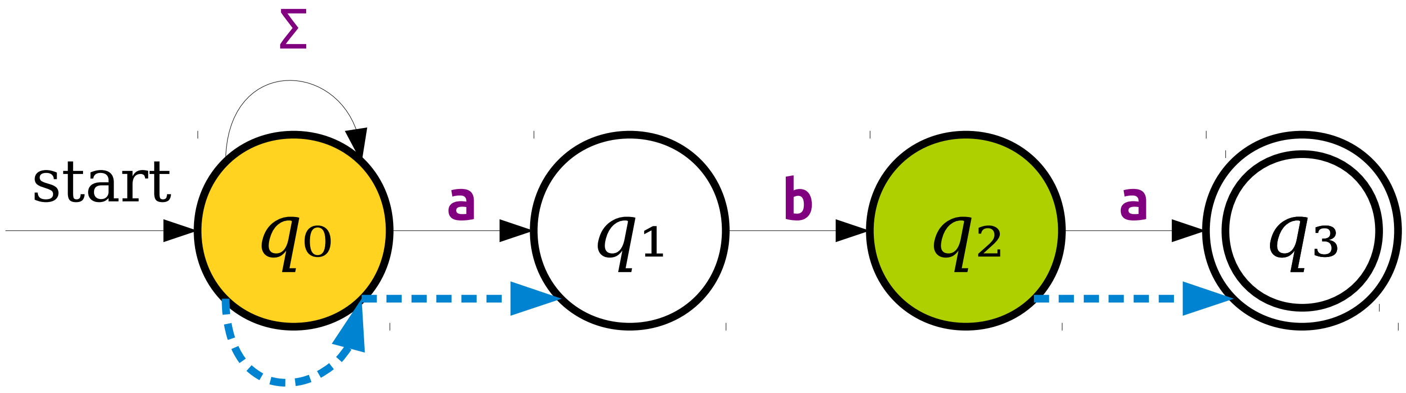

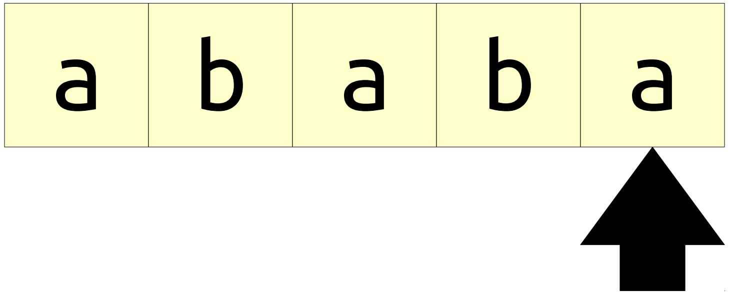

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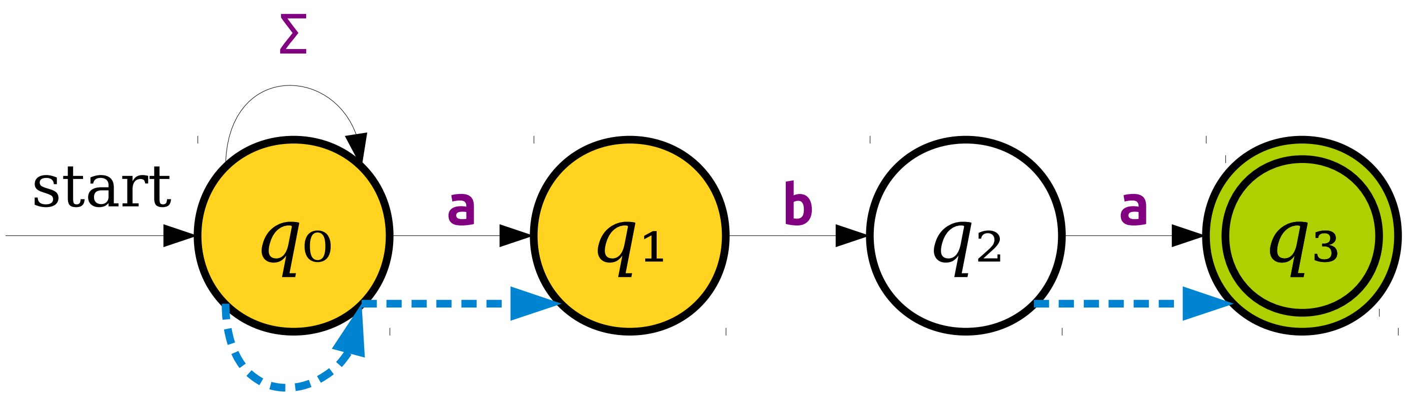

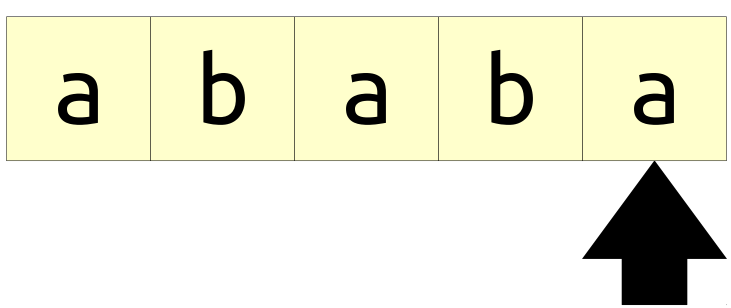

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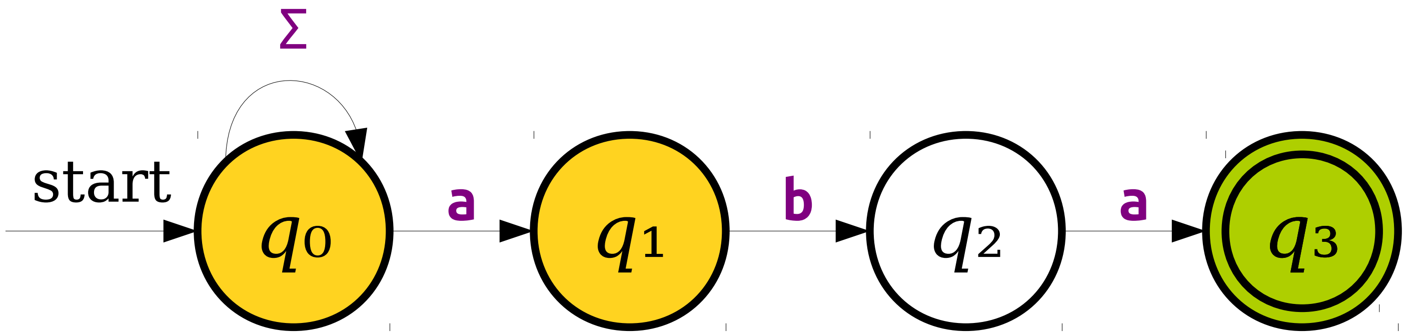

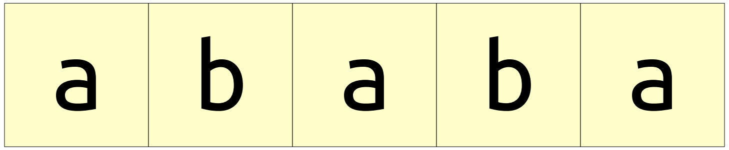

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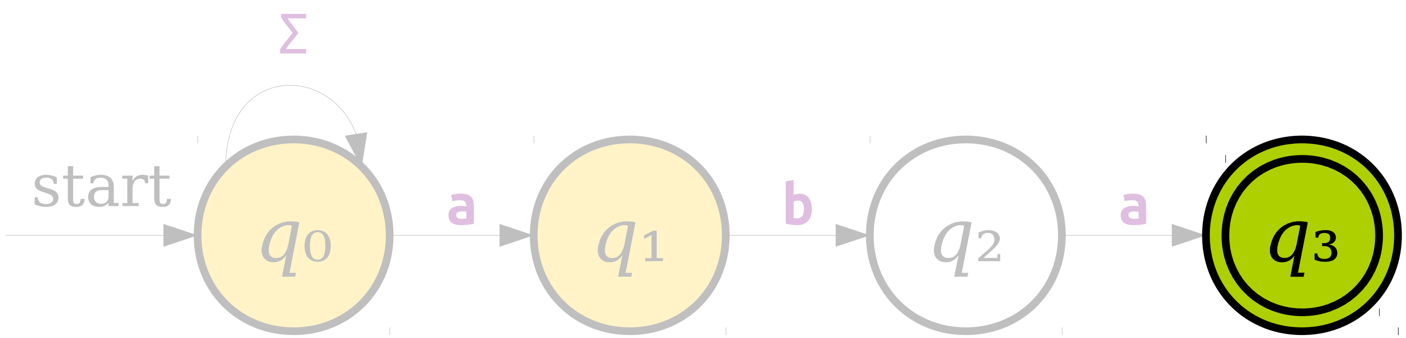
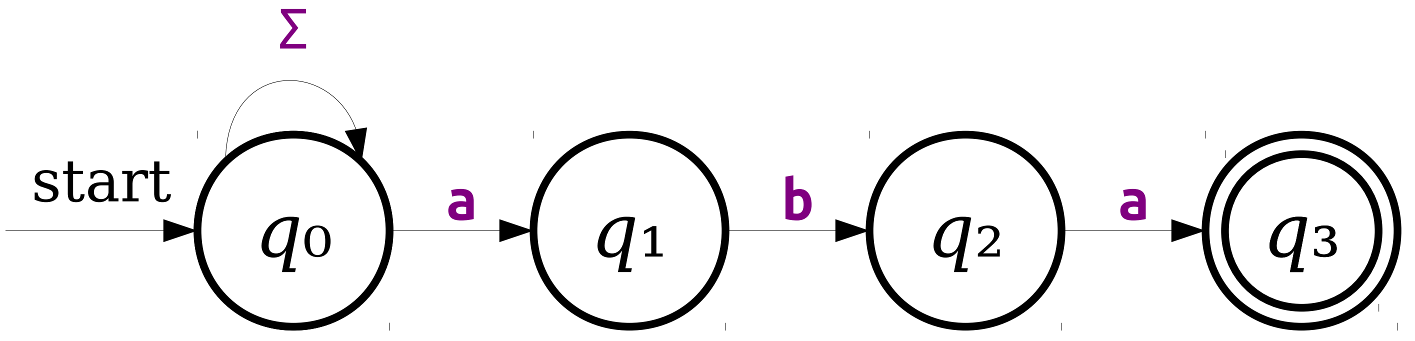
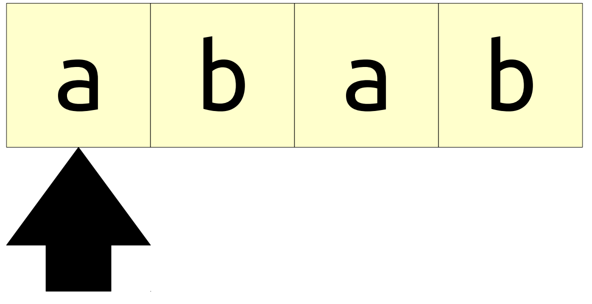


We're in at least one accepting state, so there's some path that gets us to an accepting state.

a	b	a	b	a

Massive Parallelism





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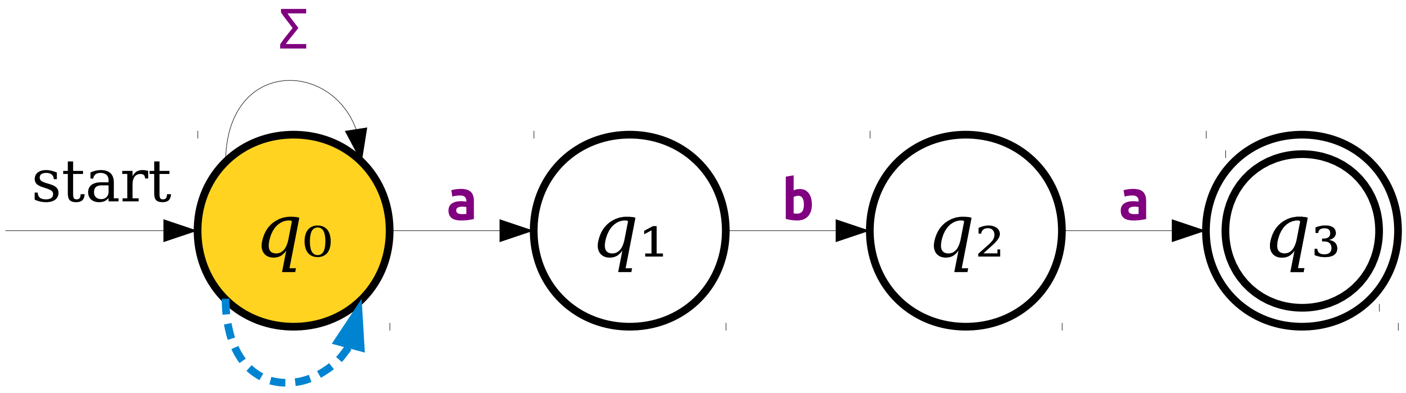

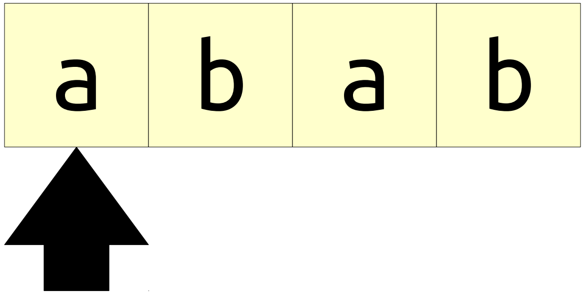

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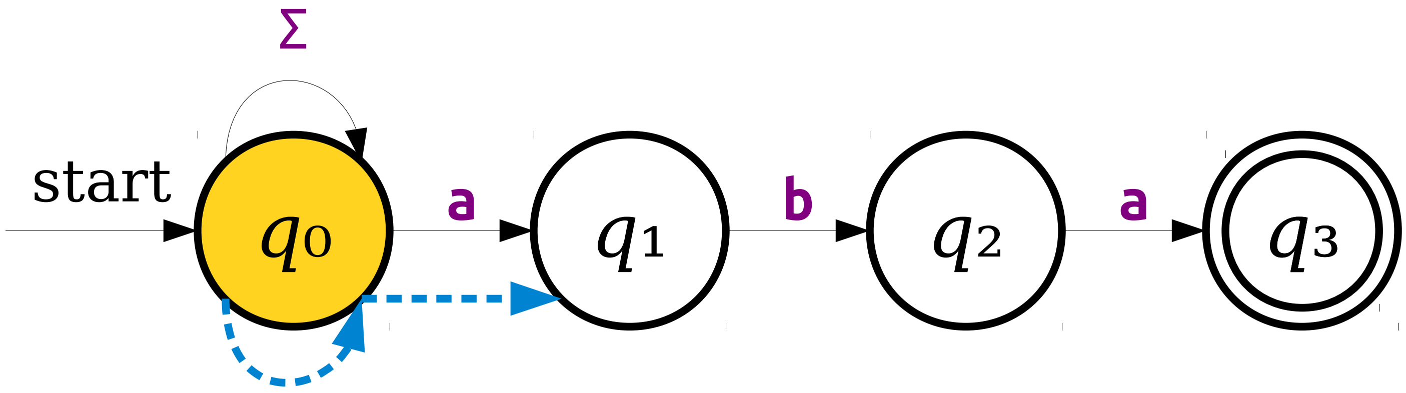

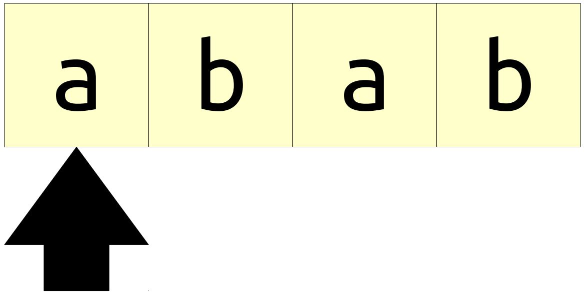

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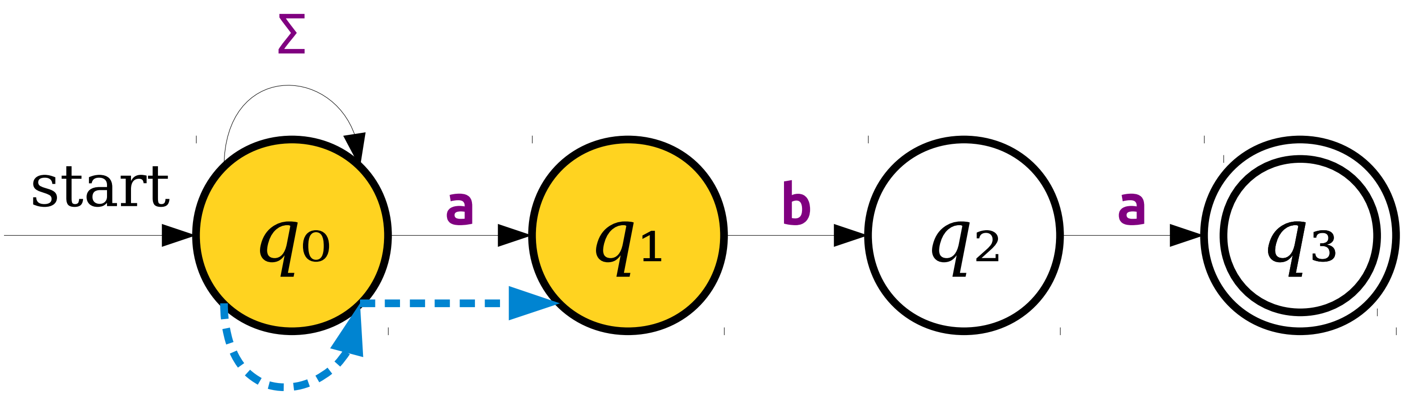

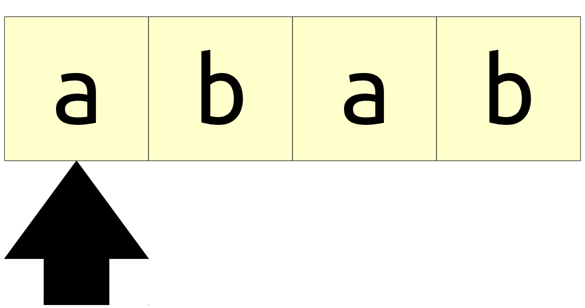

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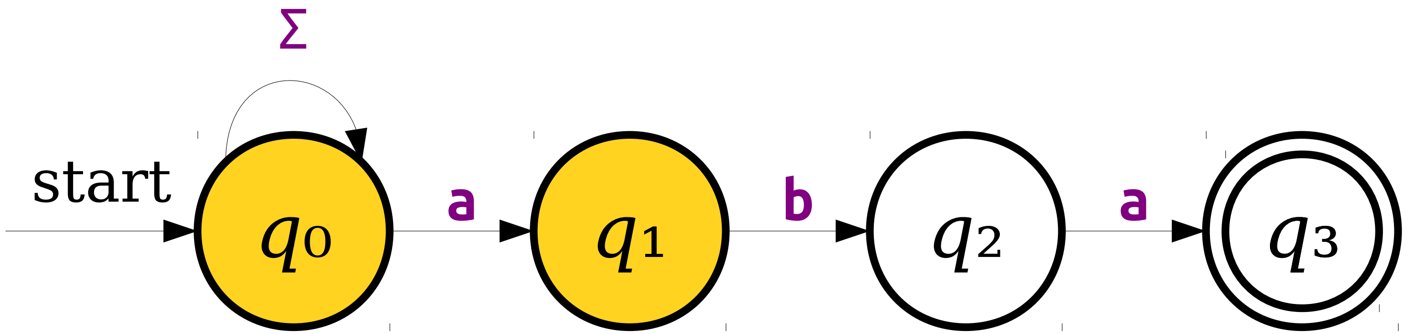

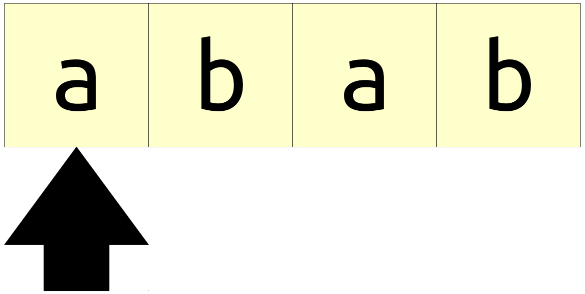

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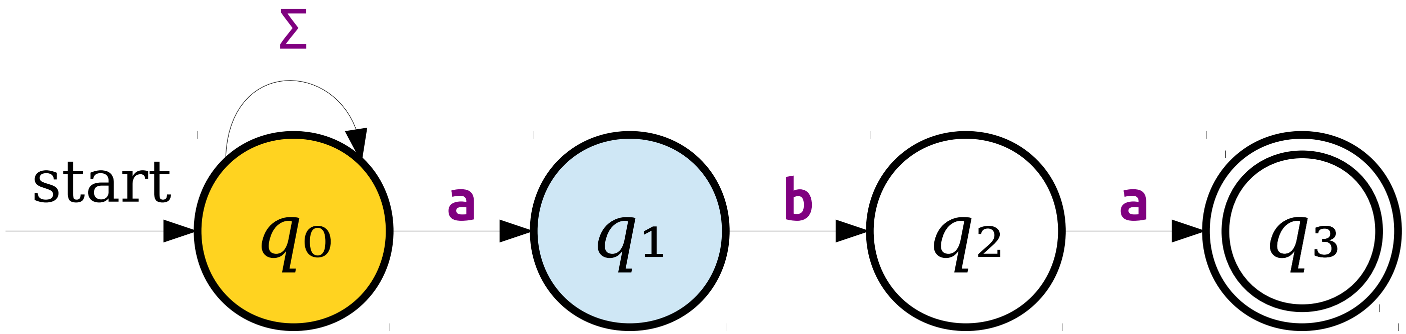

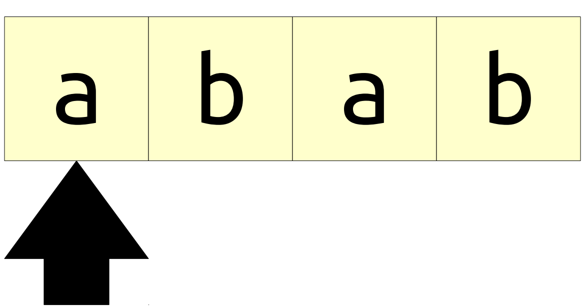

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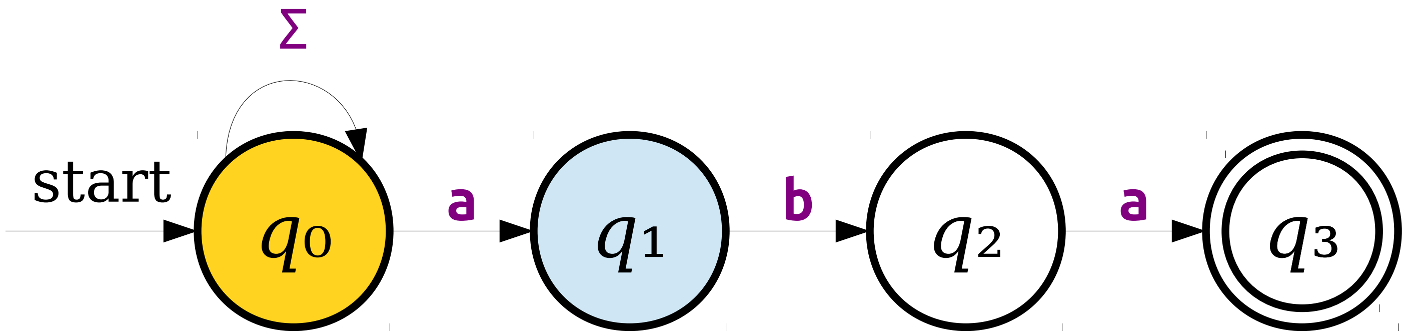

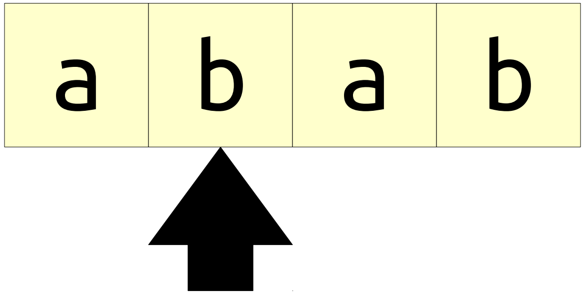

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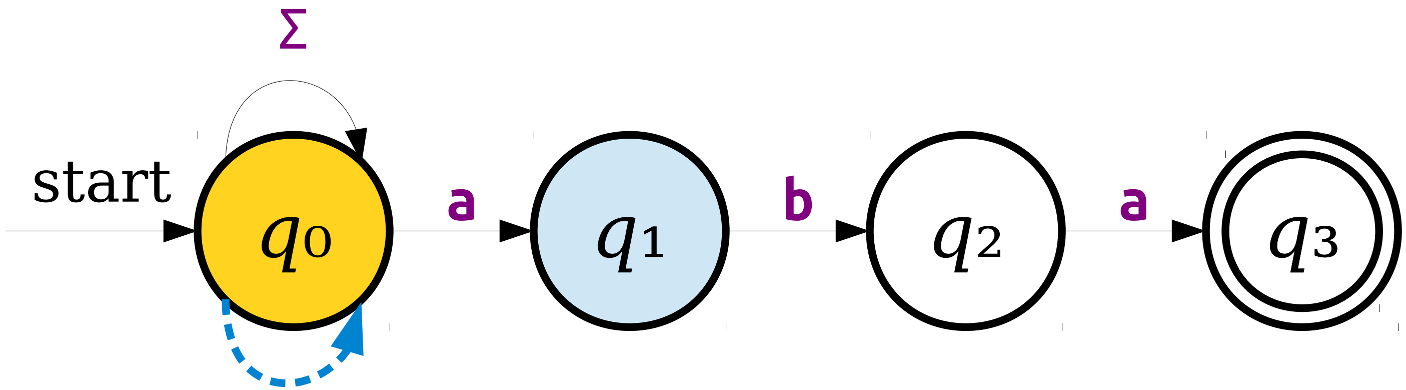

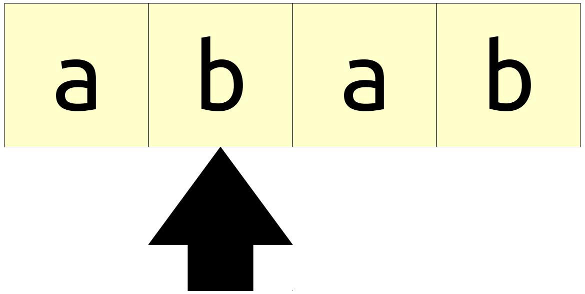

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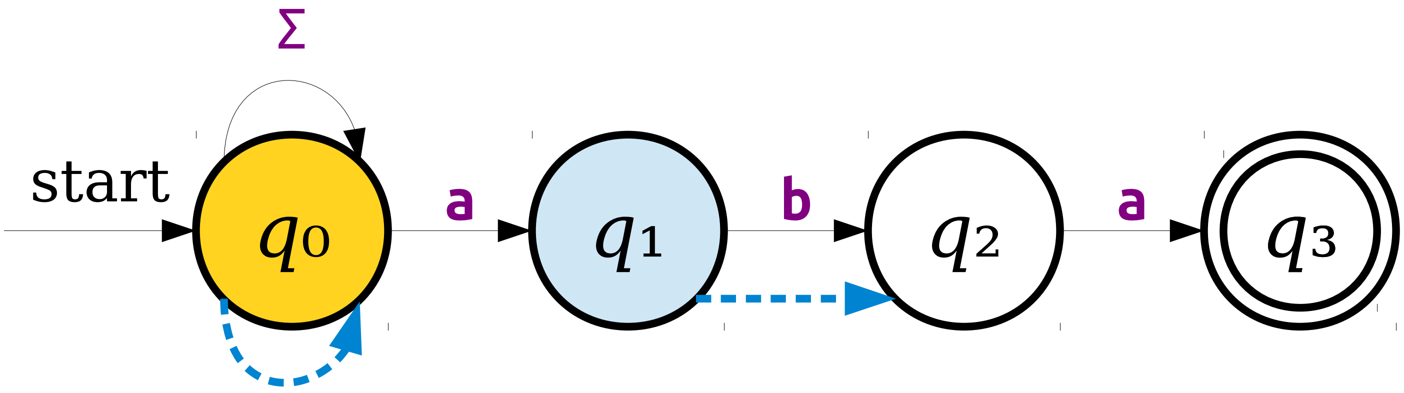

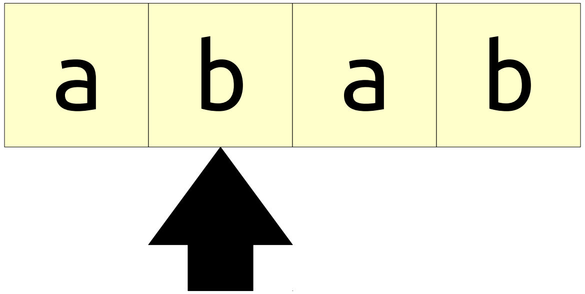

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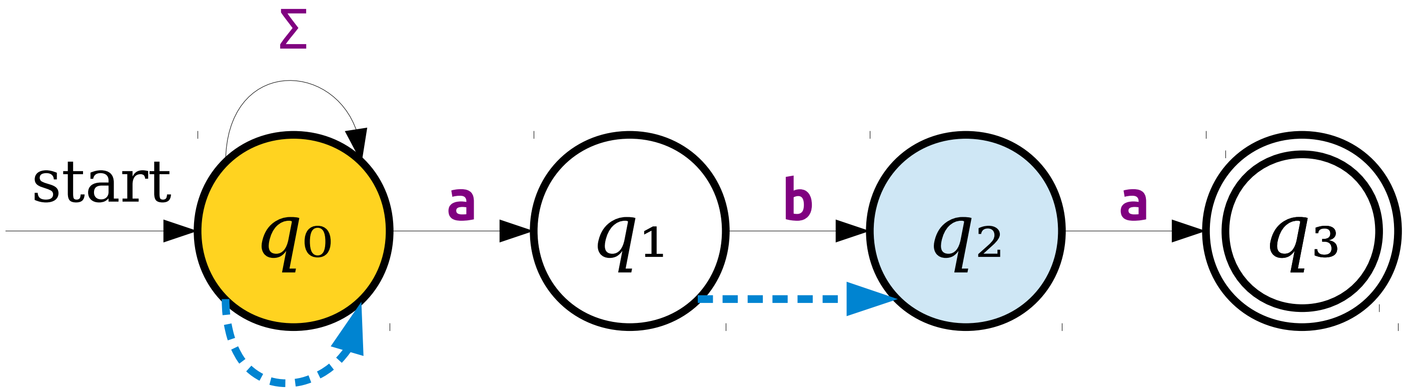

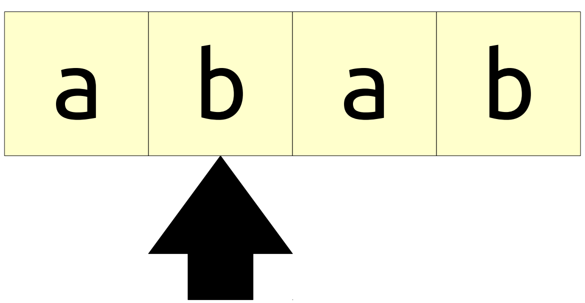

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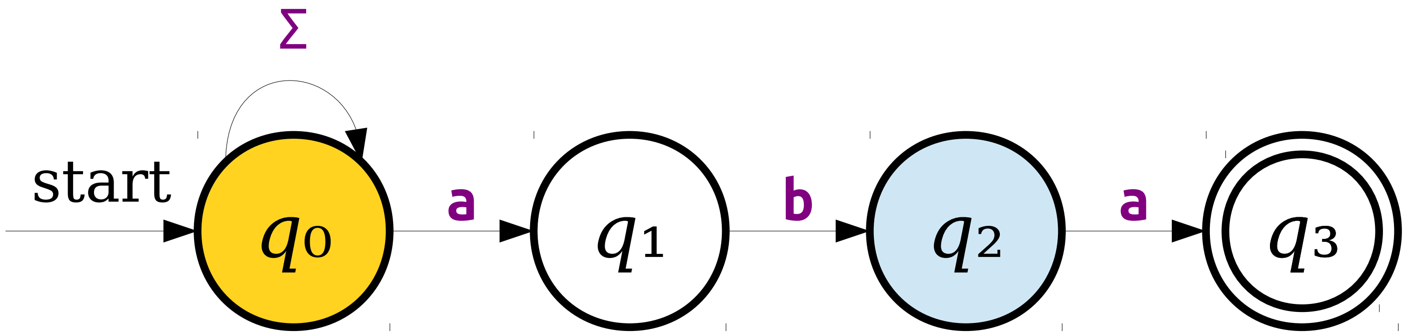

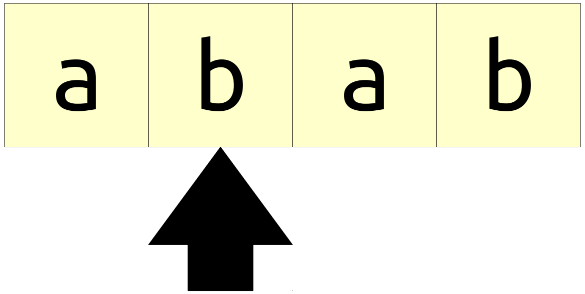

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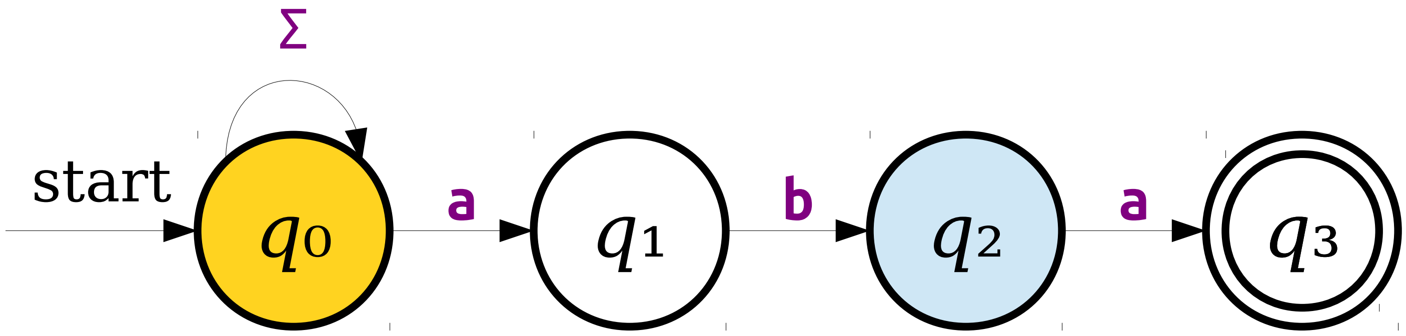

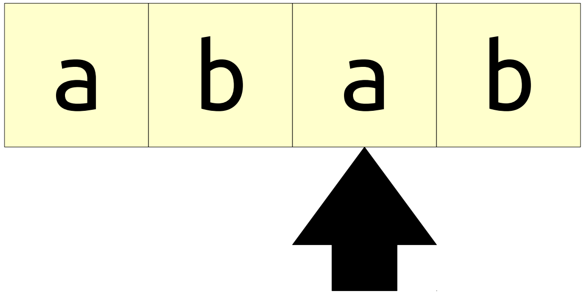

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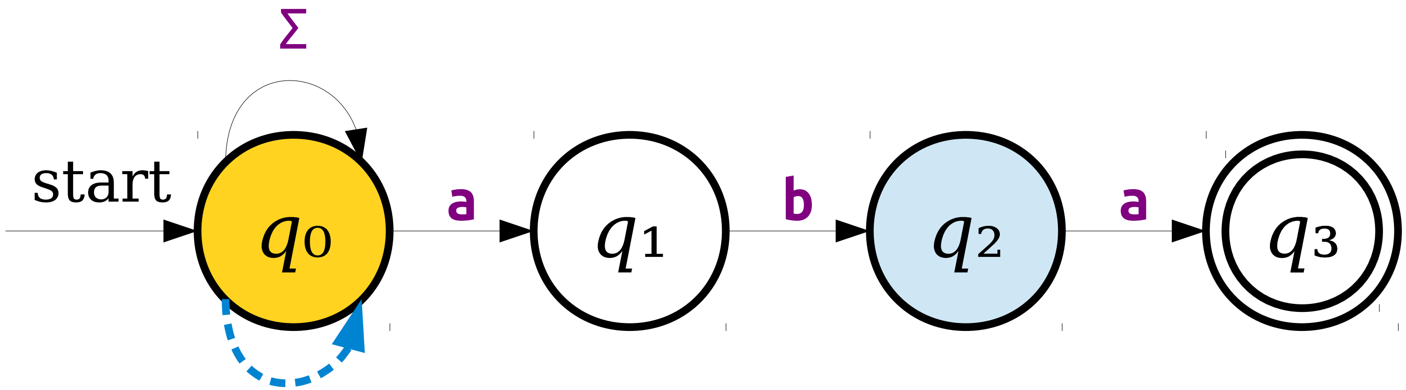

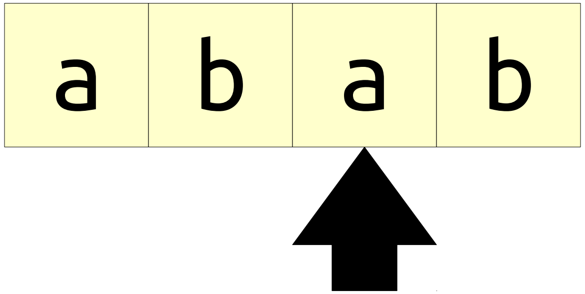

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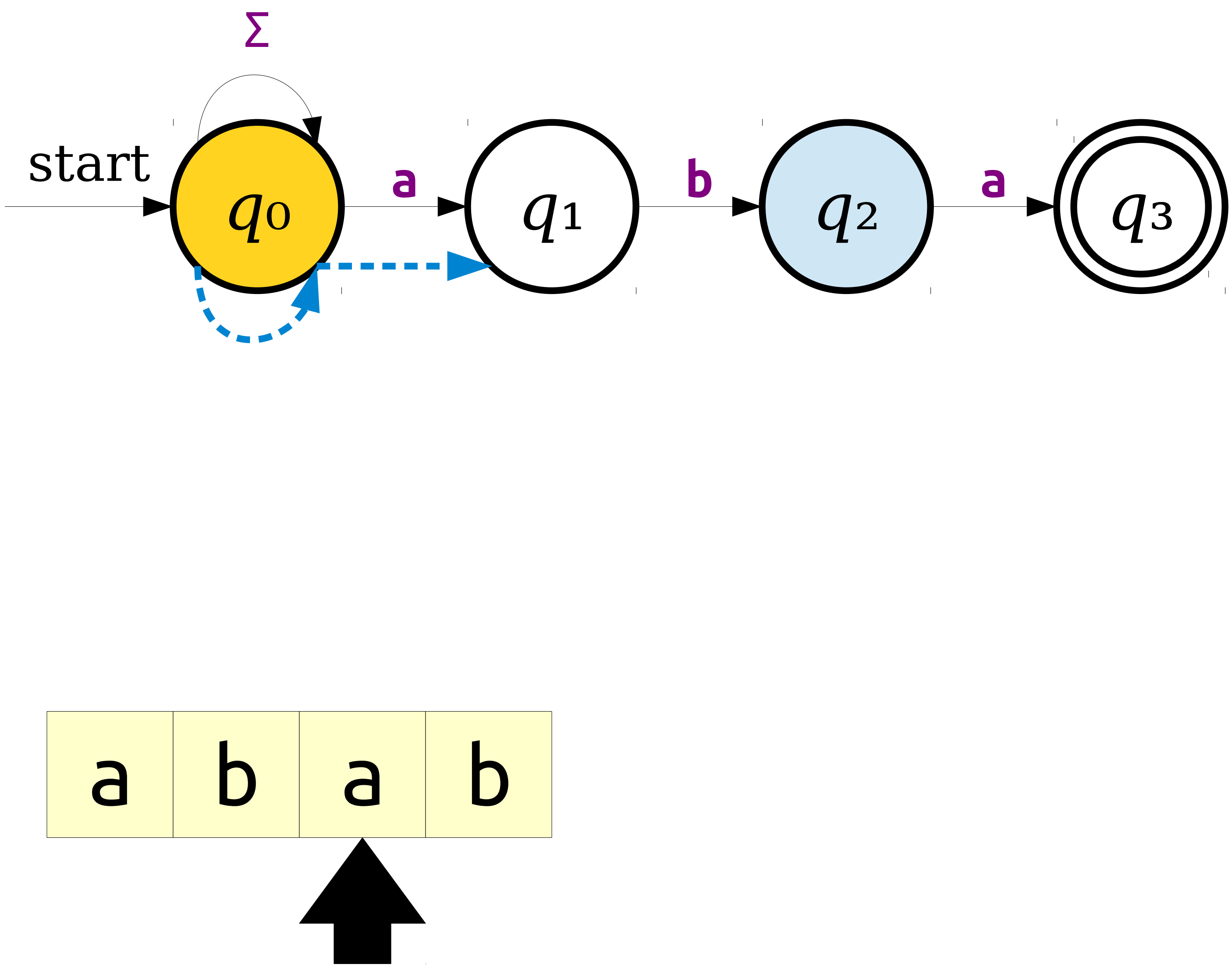

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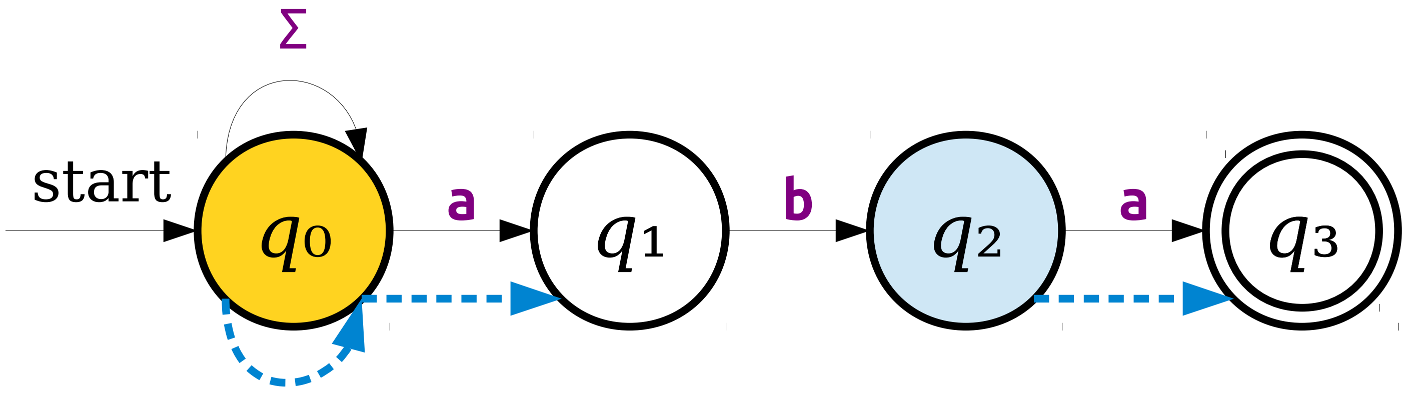

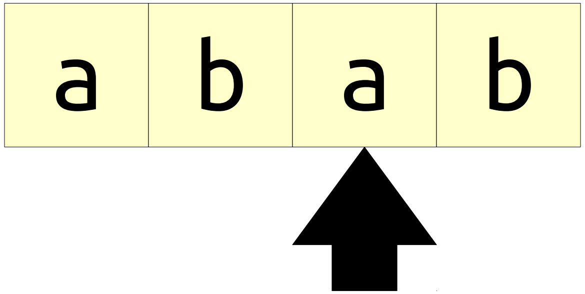

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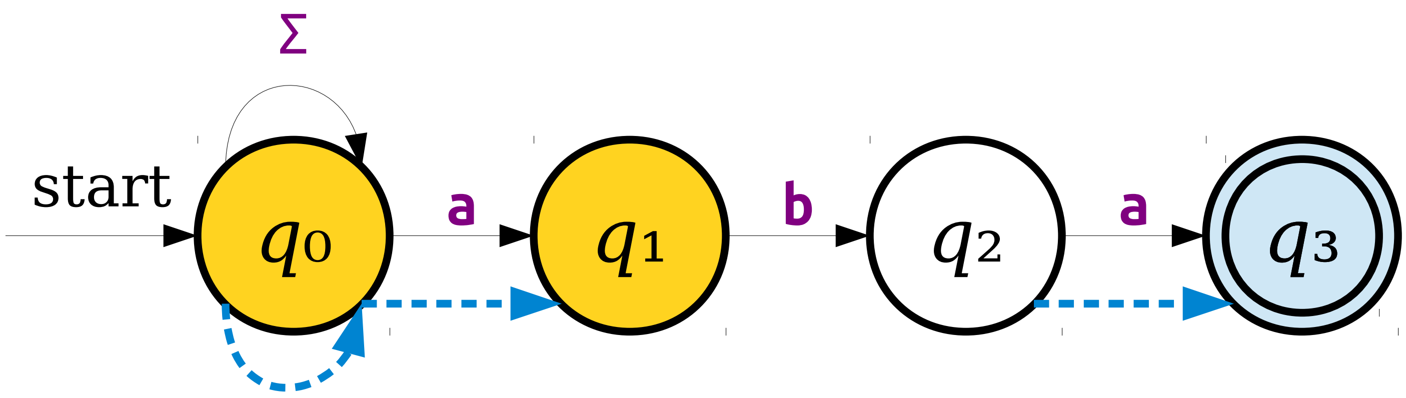

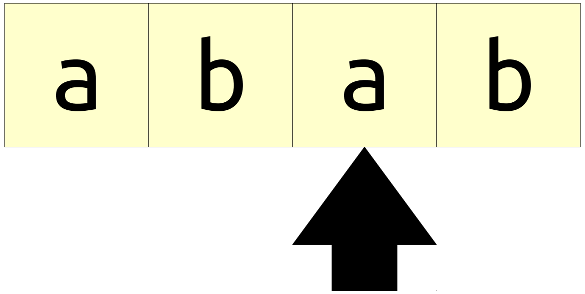

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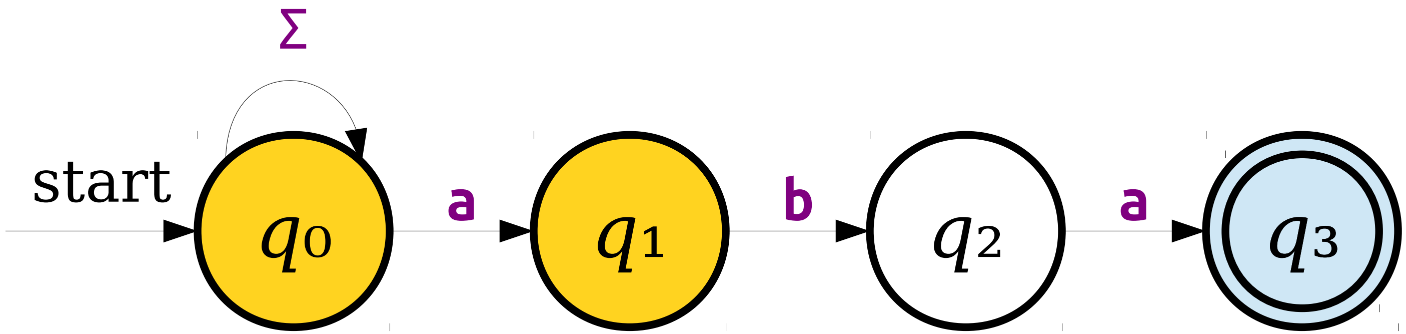

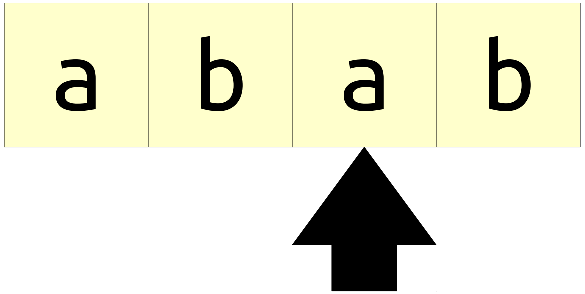

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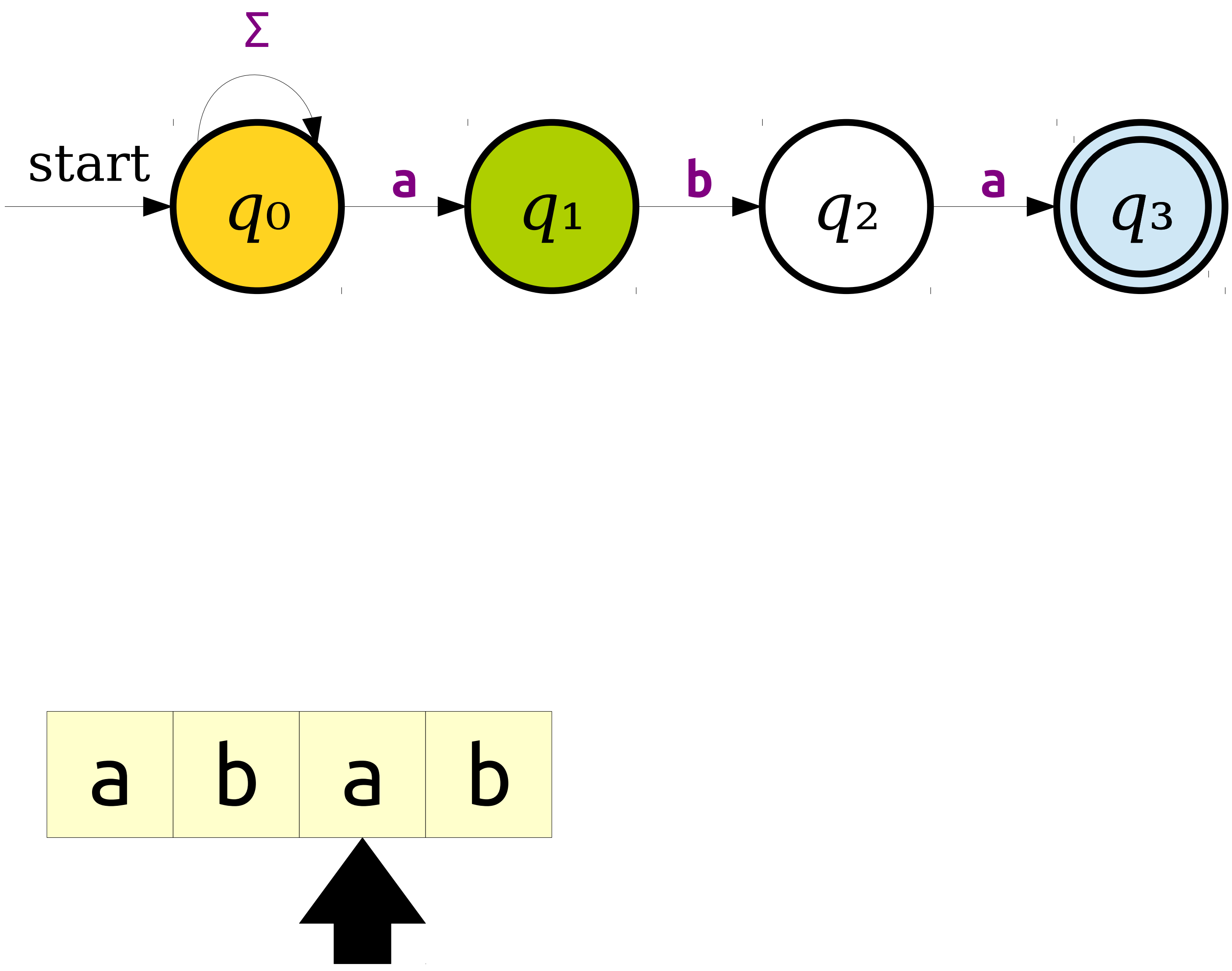

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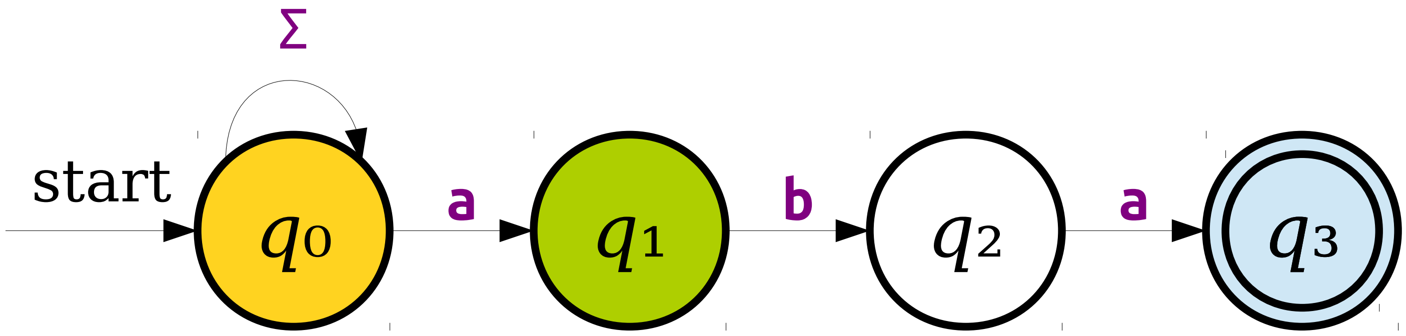

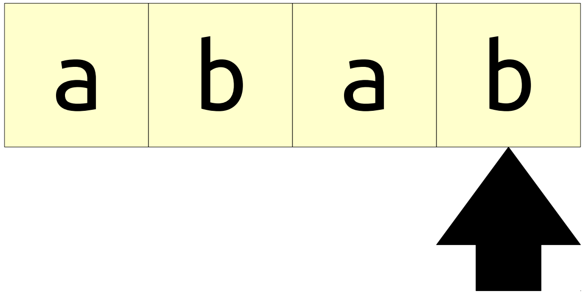

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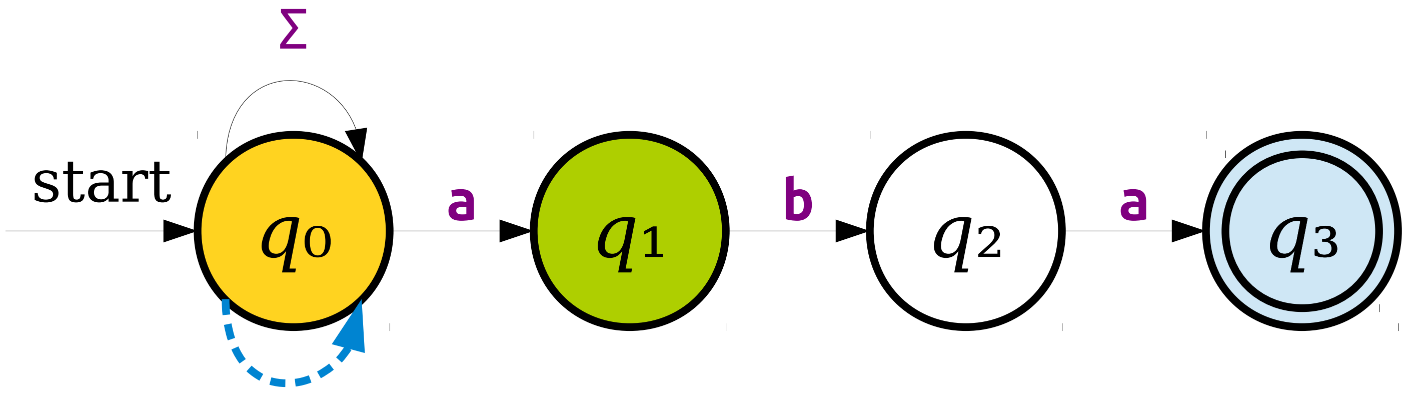

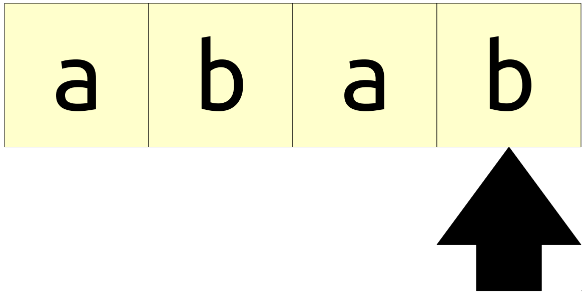

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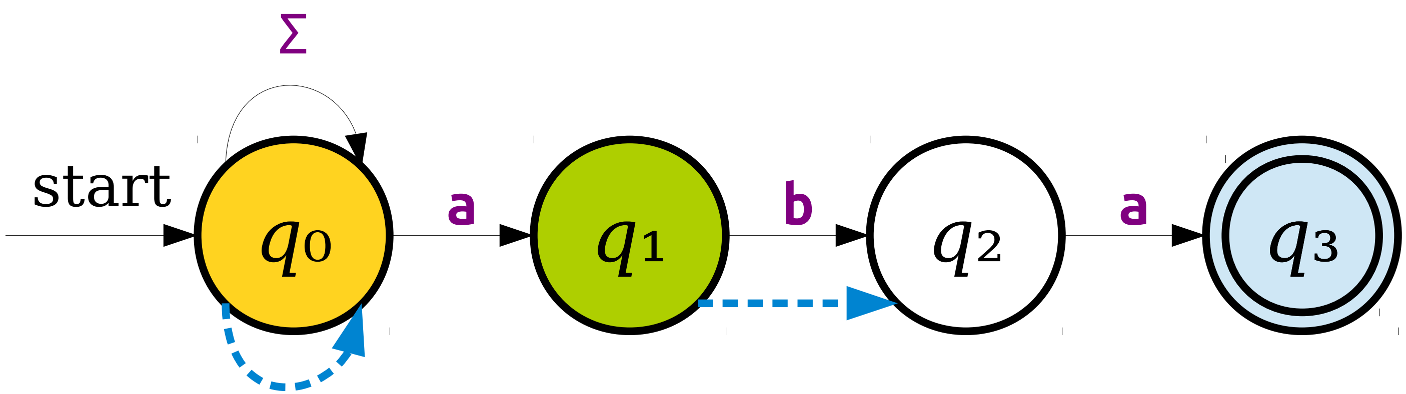

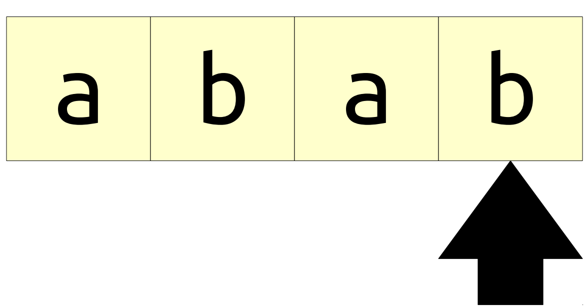

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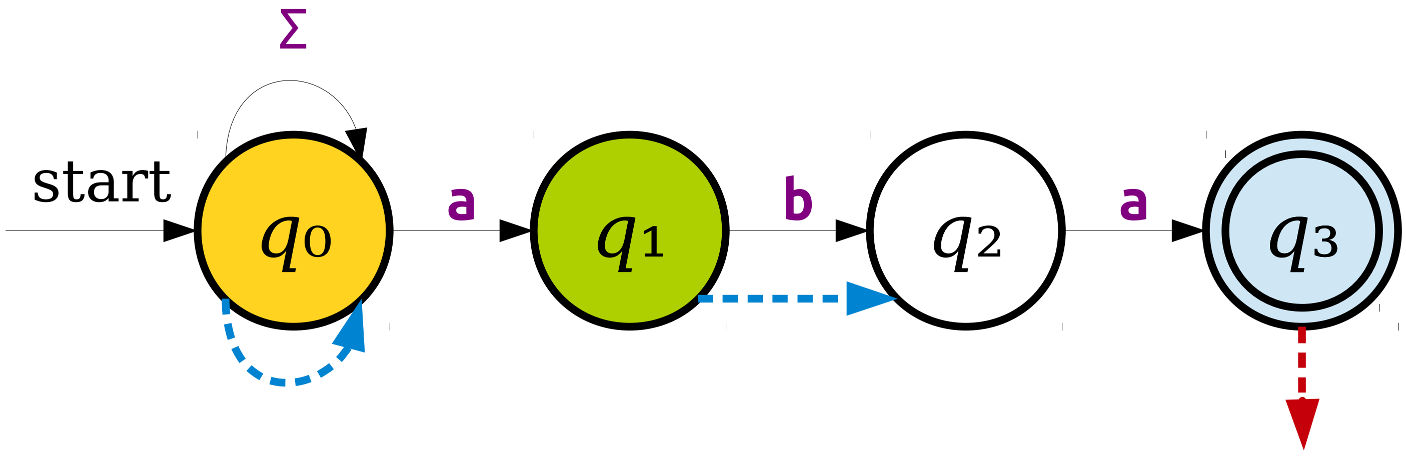

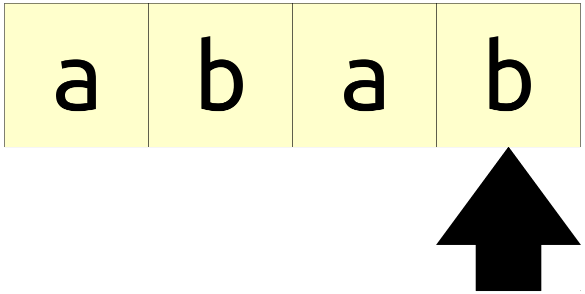

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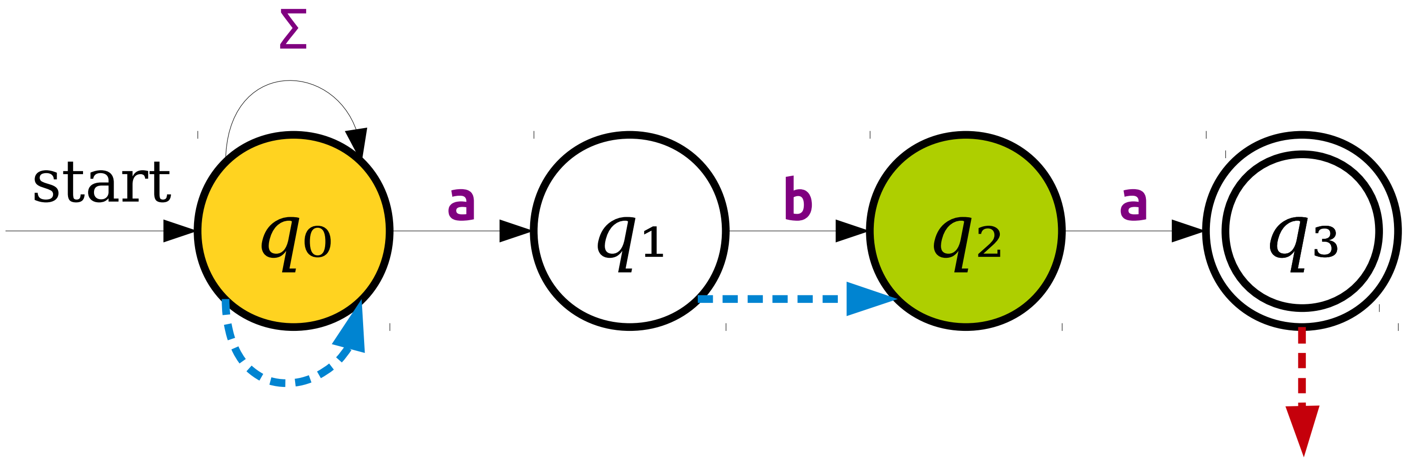

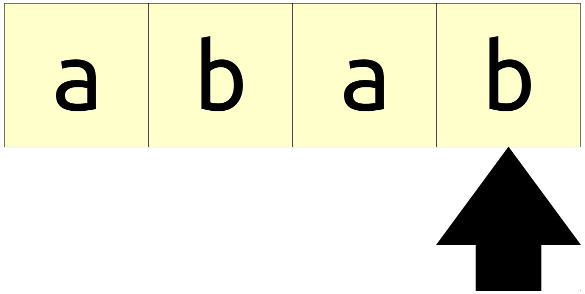

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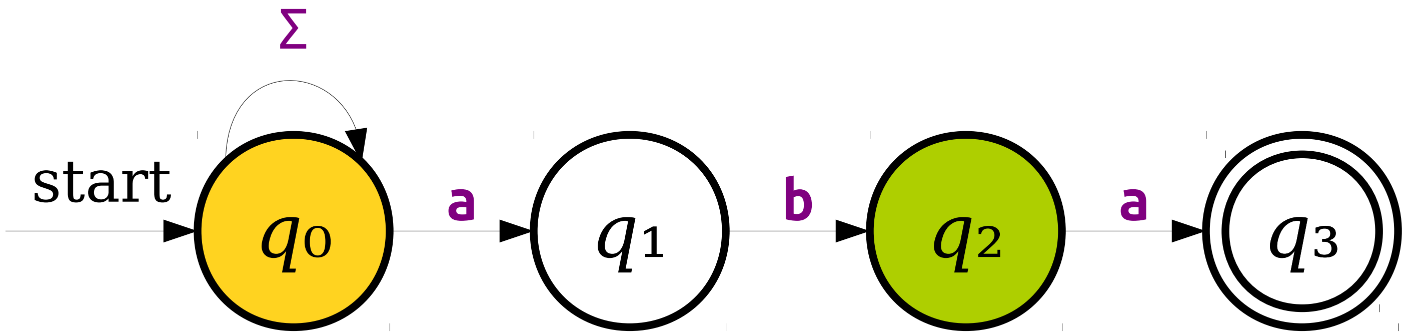

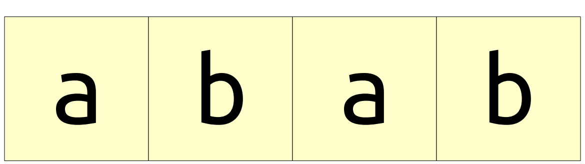

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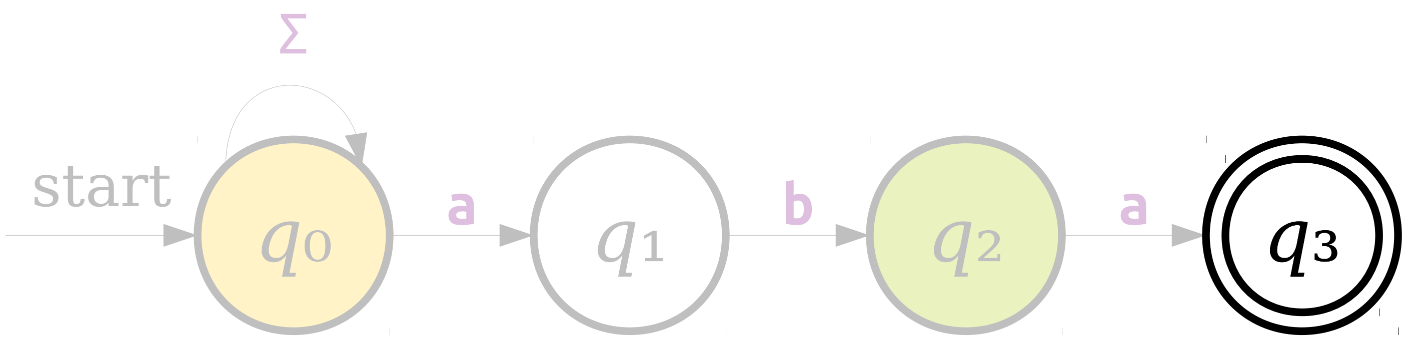


We're not in any accepting state, so no possible path accepts.

a	b	a	b

Massive Parallelism

- An NFA can be thought of as a DFA that can be in many states at once.
- At each point in time, when the NFA needs to follow a transition, it tries all the options at the same time.
- (Here's a rigorous explanation about how this works; read this on your own time).
  - Start off in the set of all states formed by taking the start state and including each state that can be reached by zero or more ε-transitions.
  - When you read a symbol **a** in a set of states *S*:
    - Form the set *S'* of states that can be reached by following a single **a** transition from some state in *S*.
    - Your new set of states is the set of states in *S'*, plus the states reachable from *S'* by following zero or more ε-transitions.

Designing NFAs

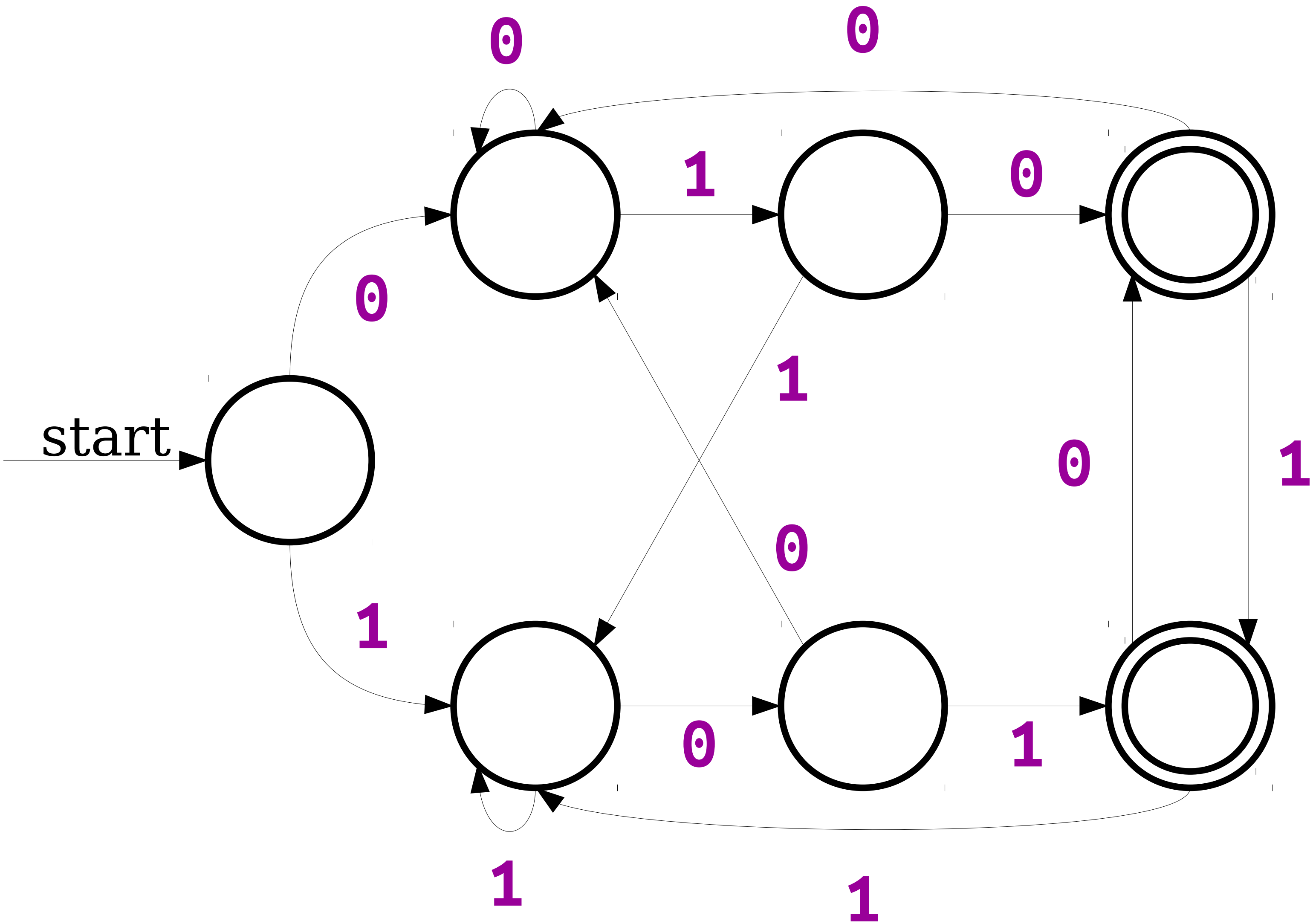
- ***Embrace the nondeterminism!***
- Good model: ***Guess-and-check***:
  - Is there some information that you'd really like to have? Have the machine *nondeterministically guess* that information.
  - Then, have the machine *deterministically check* that the choice was correct.
- The *guess* phase corresponds to trying lots of different options.
- The *check* phase corresponds to filtering out bad guesses or wrong options.

Guess-and-Check

$L = \{ w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ ends in } \mathtt{010} \text{ or } \mathtt{101} \}$

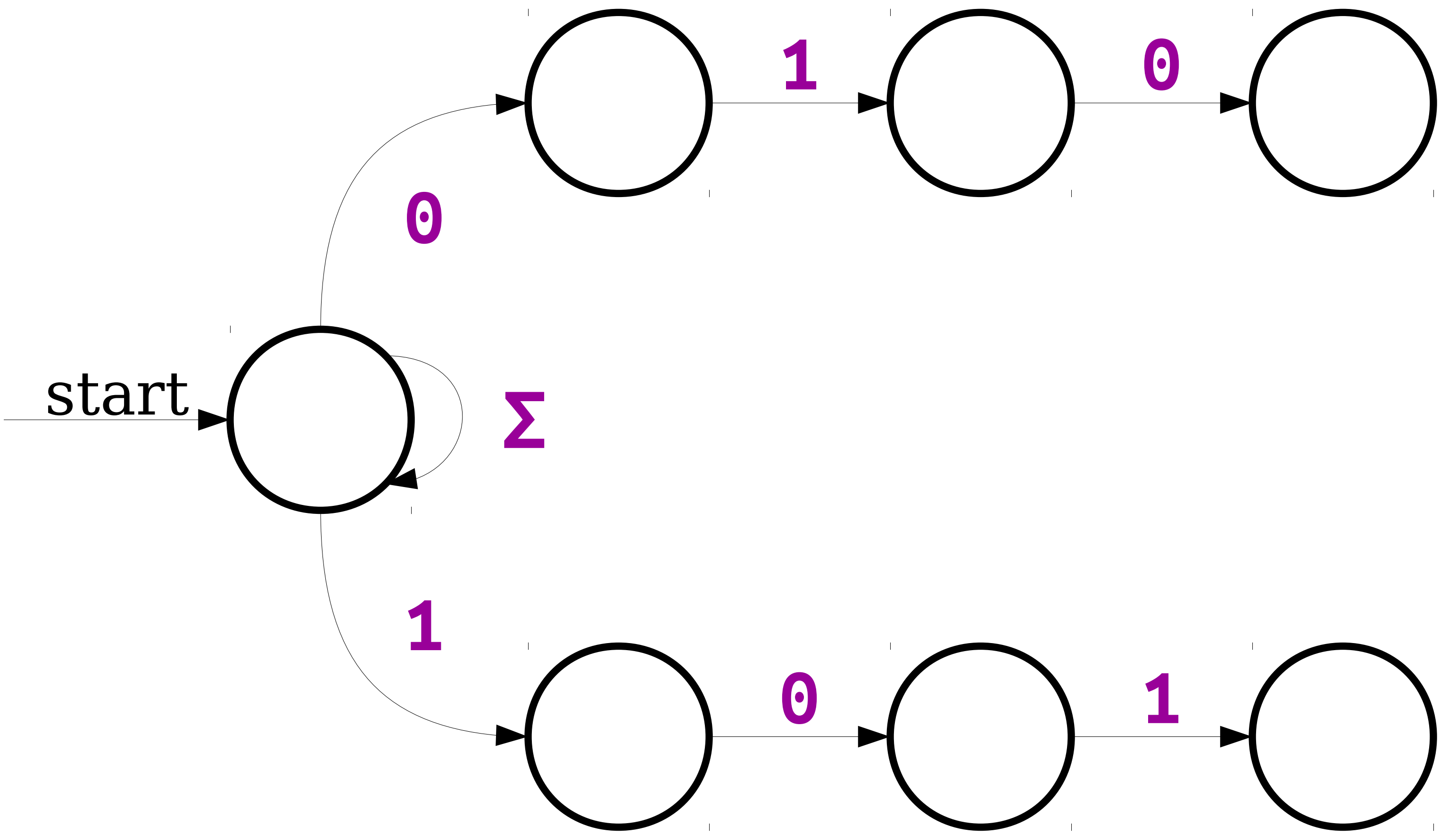
Guess-and-Check

$L = \{\ w \in \{$**0**, **1**$\}^* \mid w$ ends in **010** or **101** $\}$



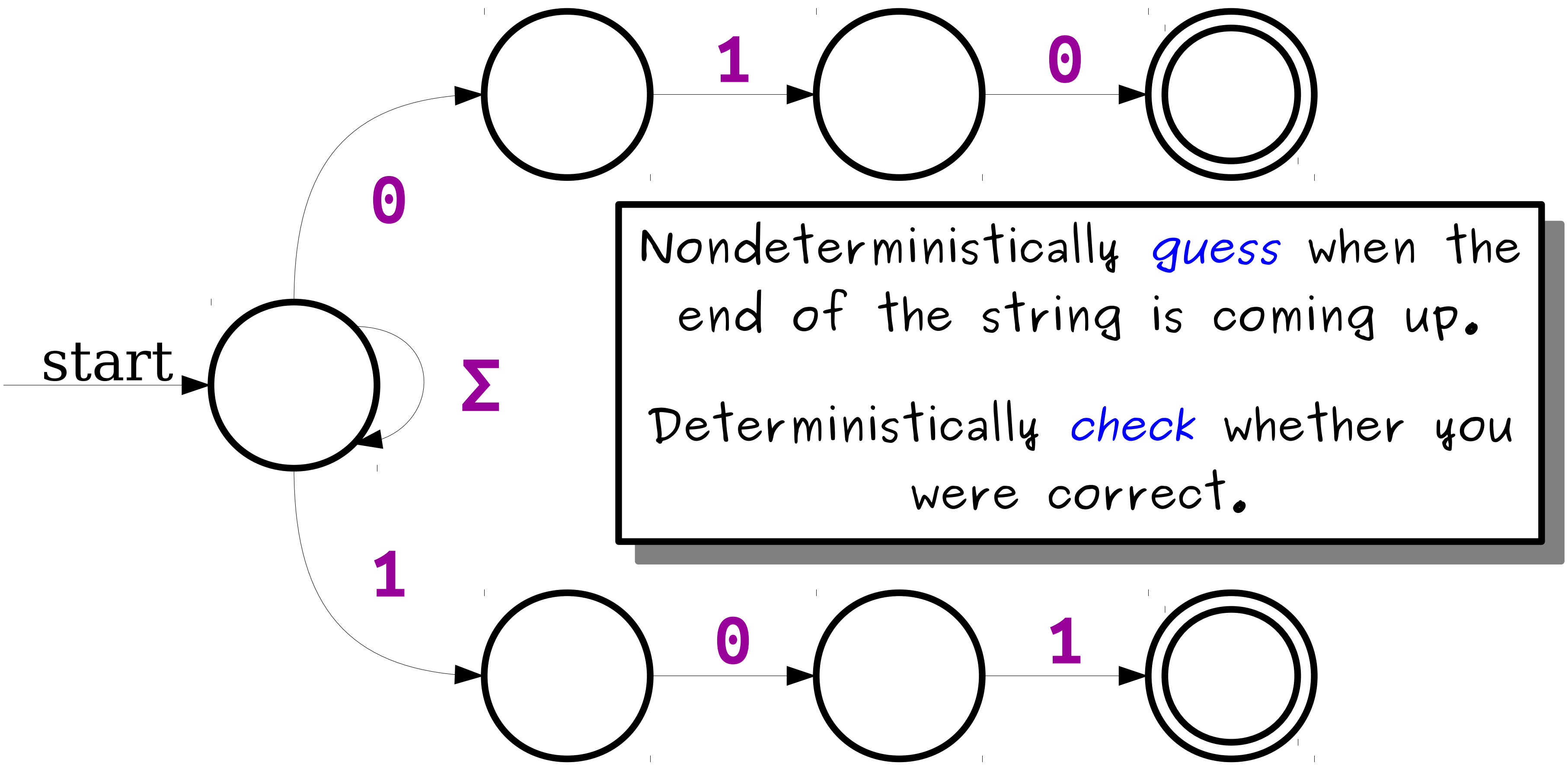
Guess-and-Check

$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$



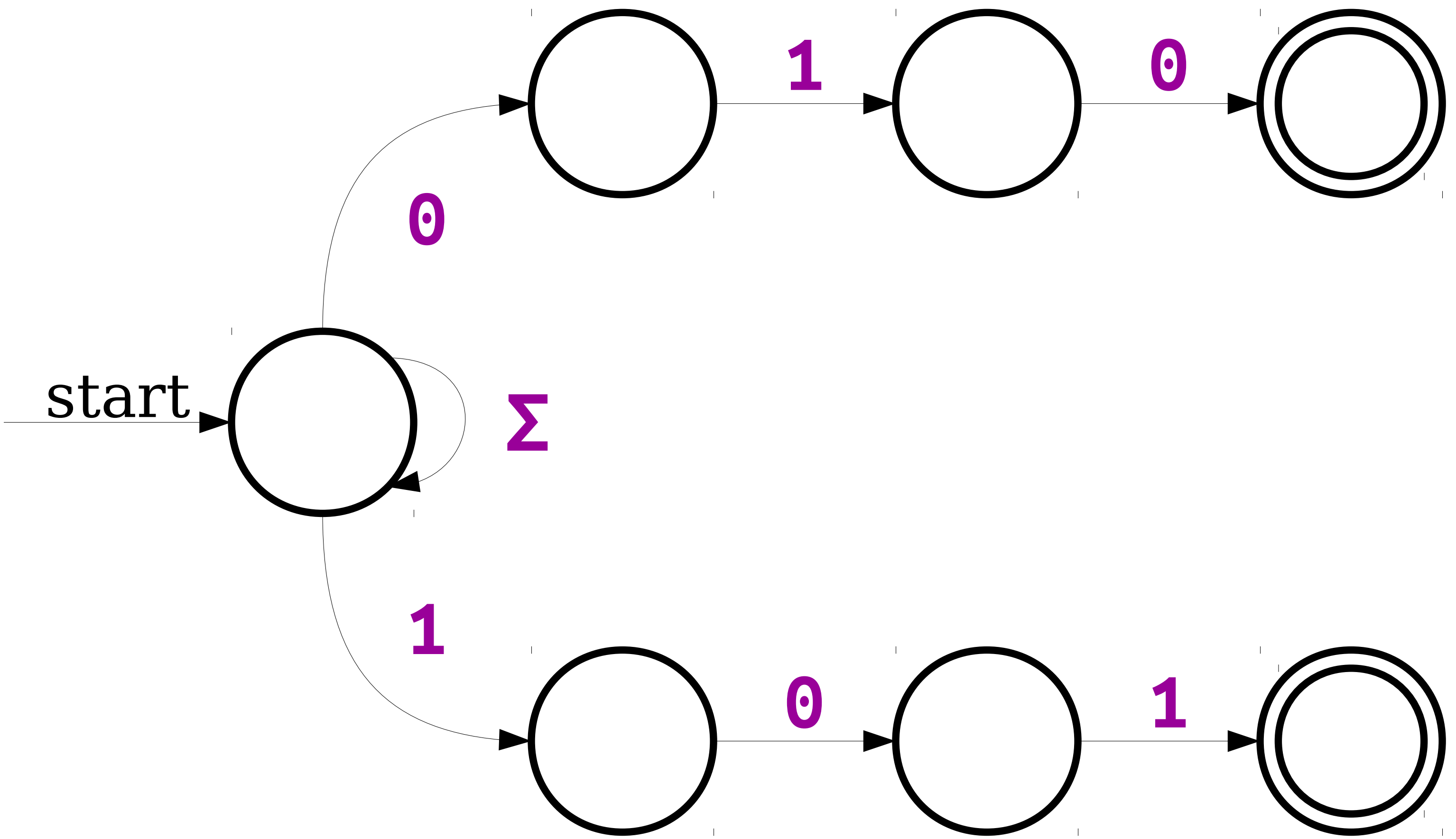
Guess-and-Check

$$L = \{\ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ ends in } \texttt{010} \text{ or } \texttt{101}\ \}$$



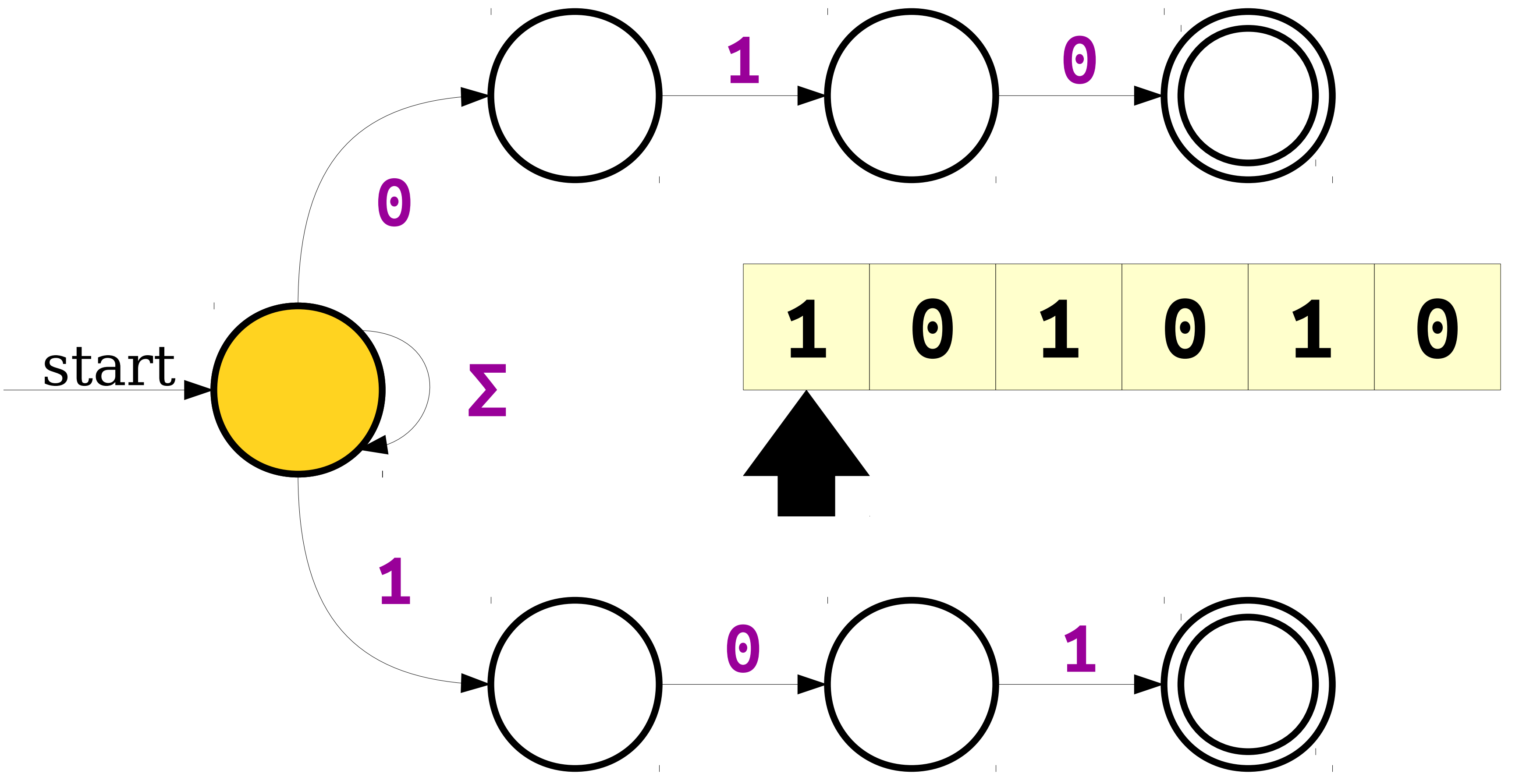
Guess-and-Check

$L = \{ w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ ends in } \mathtt{010} \text{ or } \mathtt{101} \}$



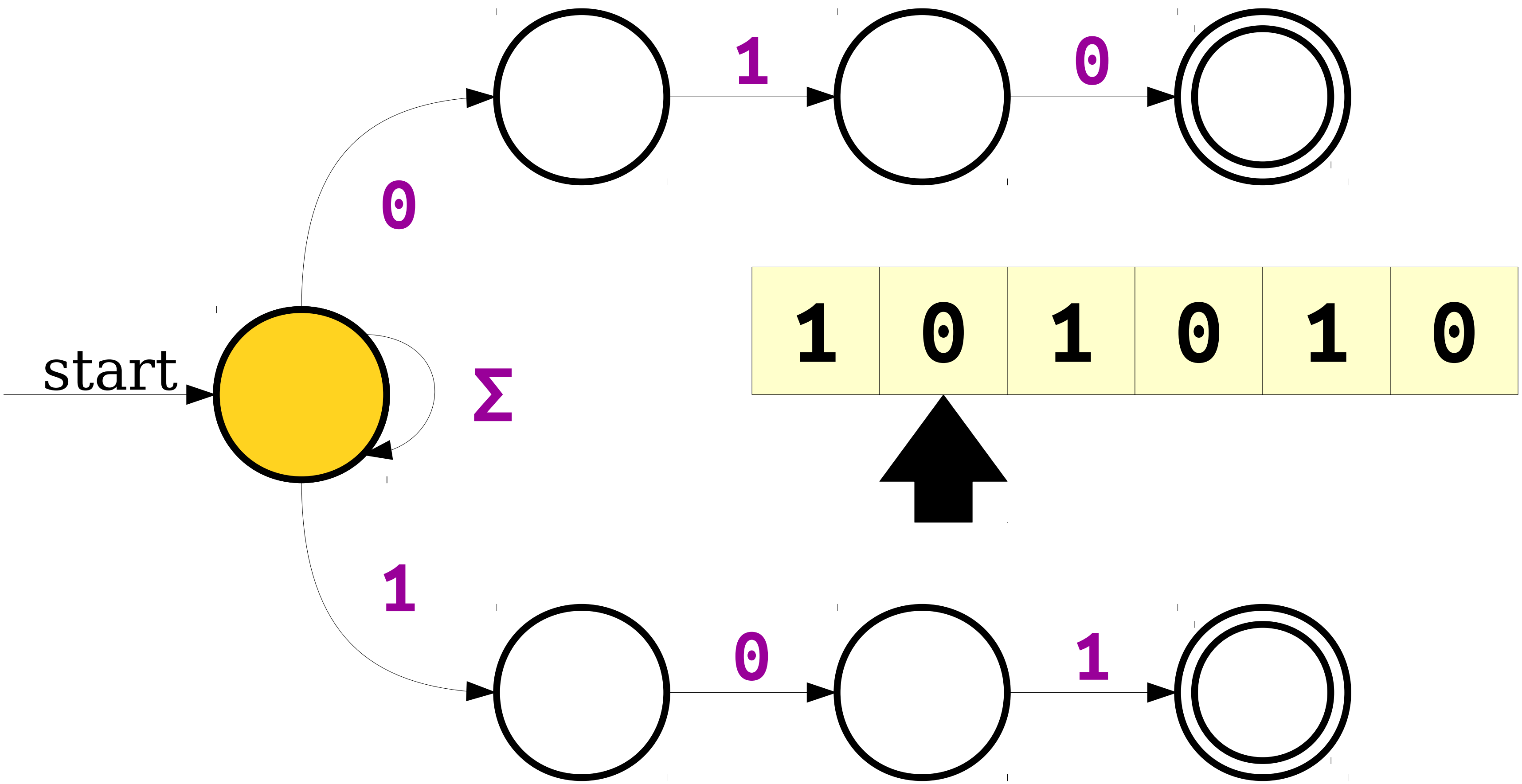
Guess-and-Check

$L = \{ w \in \{$**0**, **1**$\}^* \mid w$ ends in **010** or **101** $\}$



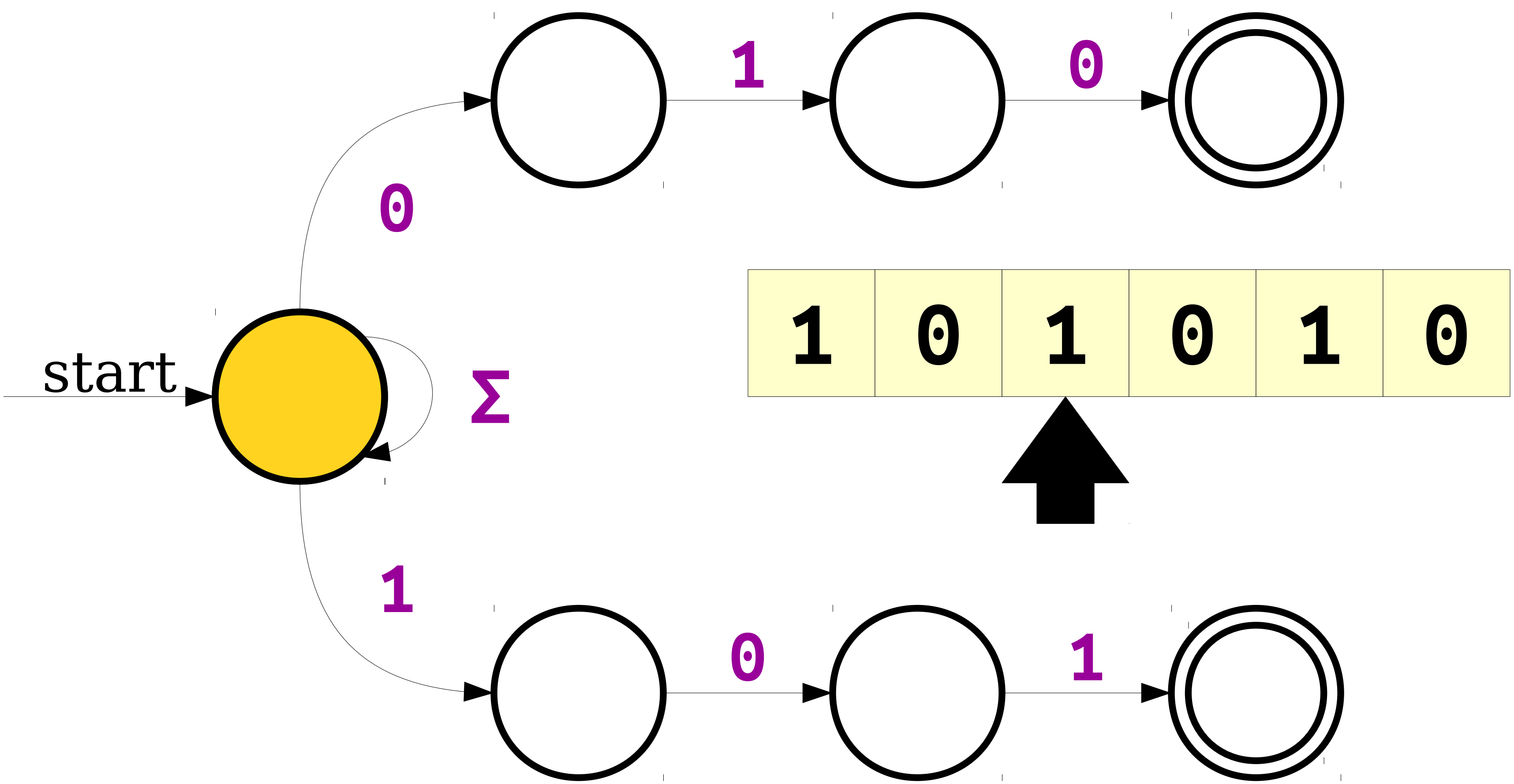
Guess-and-Check

$L = \{ w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ ends in } \mathtt{010} \text{ or } \mathtt{101} \}$



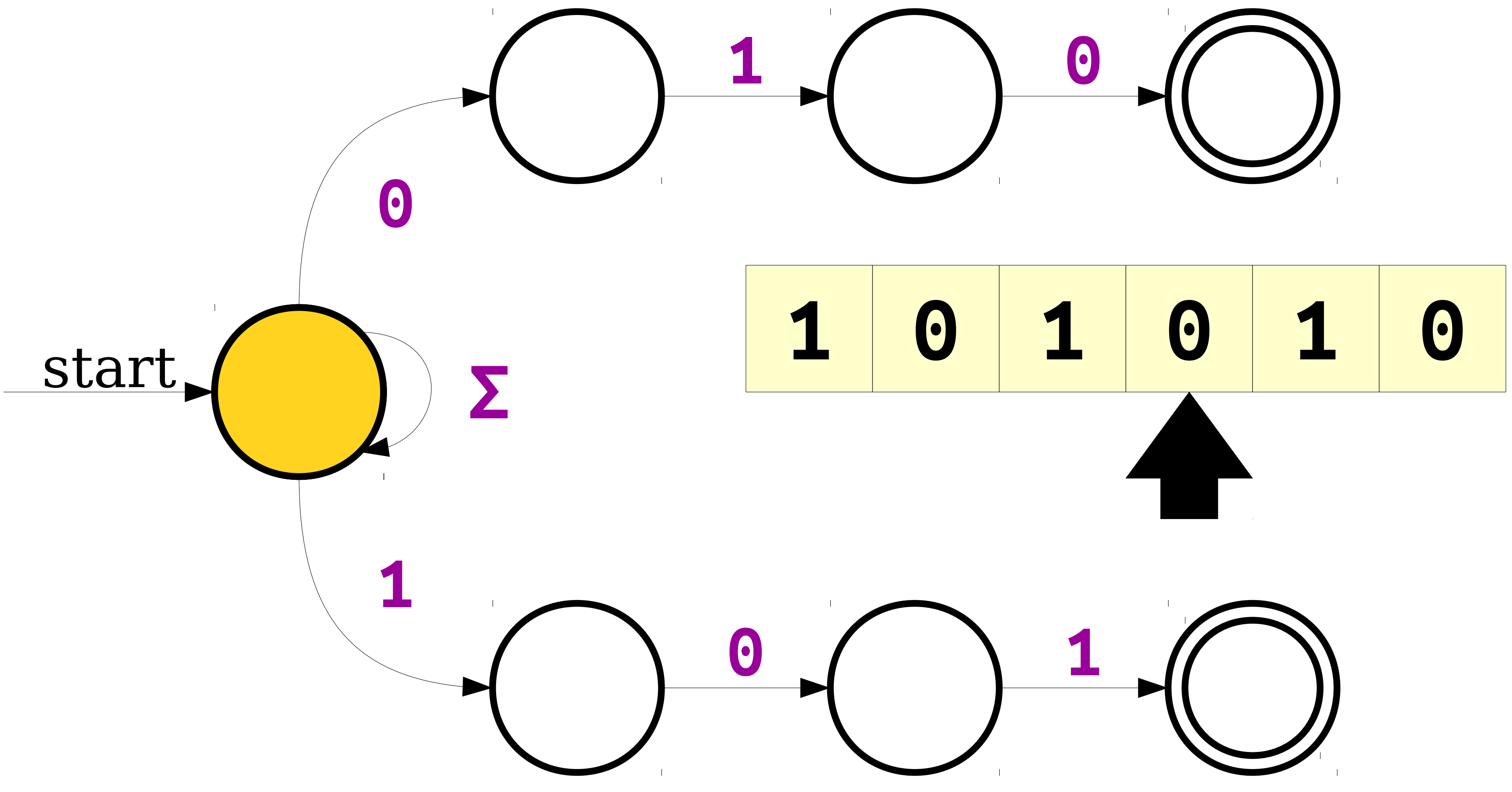
Guess-and-Check

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ ends in } \texttt{010} \text{ or } \texttt{101} \}$



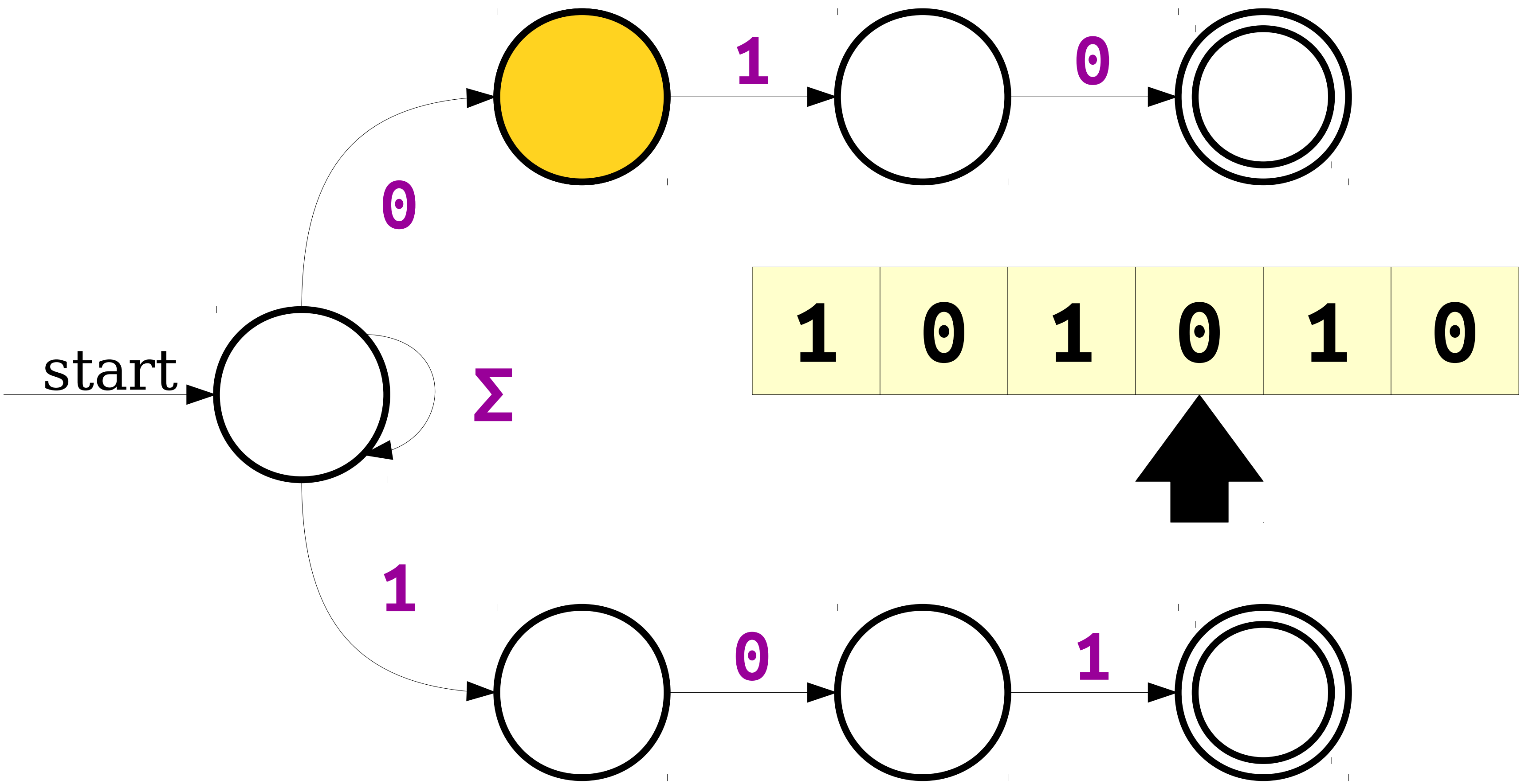
Guess-and-Check

$L = \{ w \in \{\mathtt{0}, \mathtt{1}\}^* \mid w \text{ ends in } \mathtt{010} \text{ or } \mathtt{101} \}$



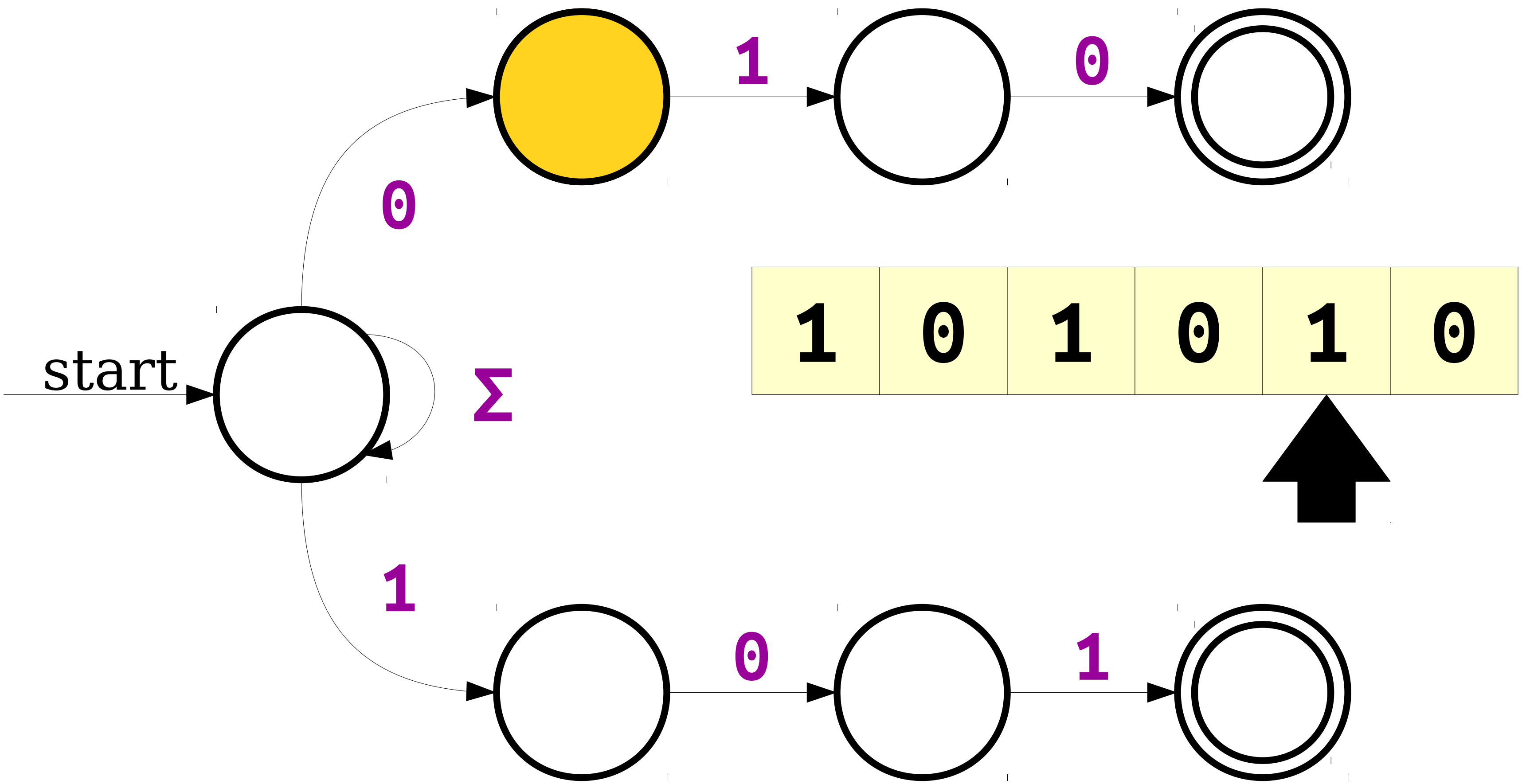
Guess-and-Check

$L = \{ w \in \{$**0**, **1**$\}^* \mid w$ ends in **010** or **101** $\}$



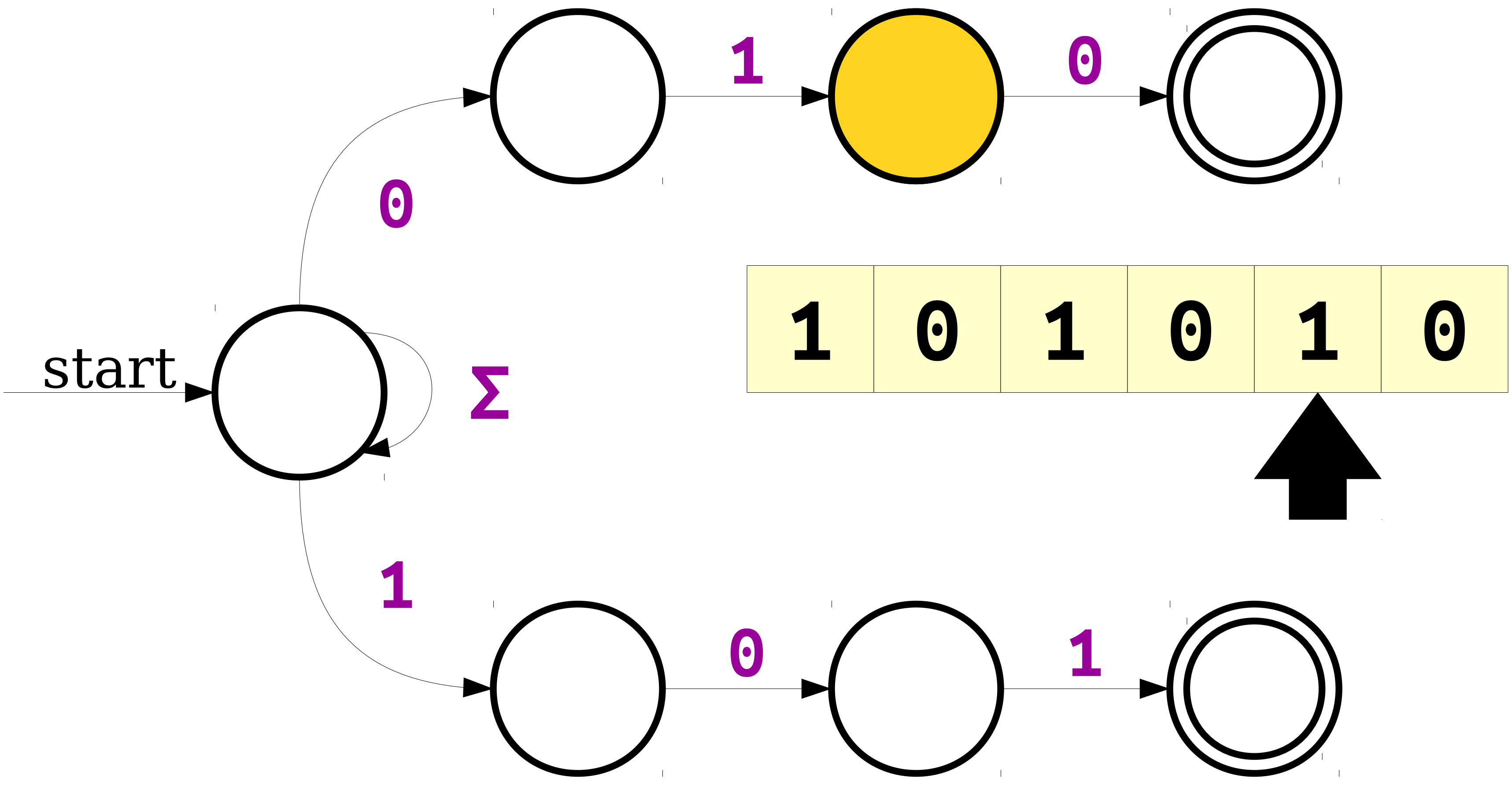
Guess-and-Check

$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$



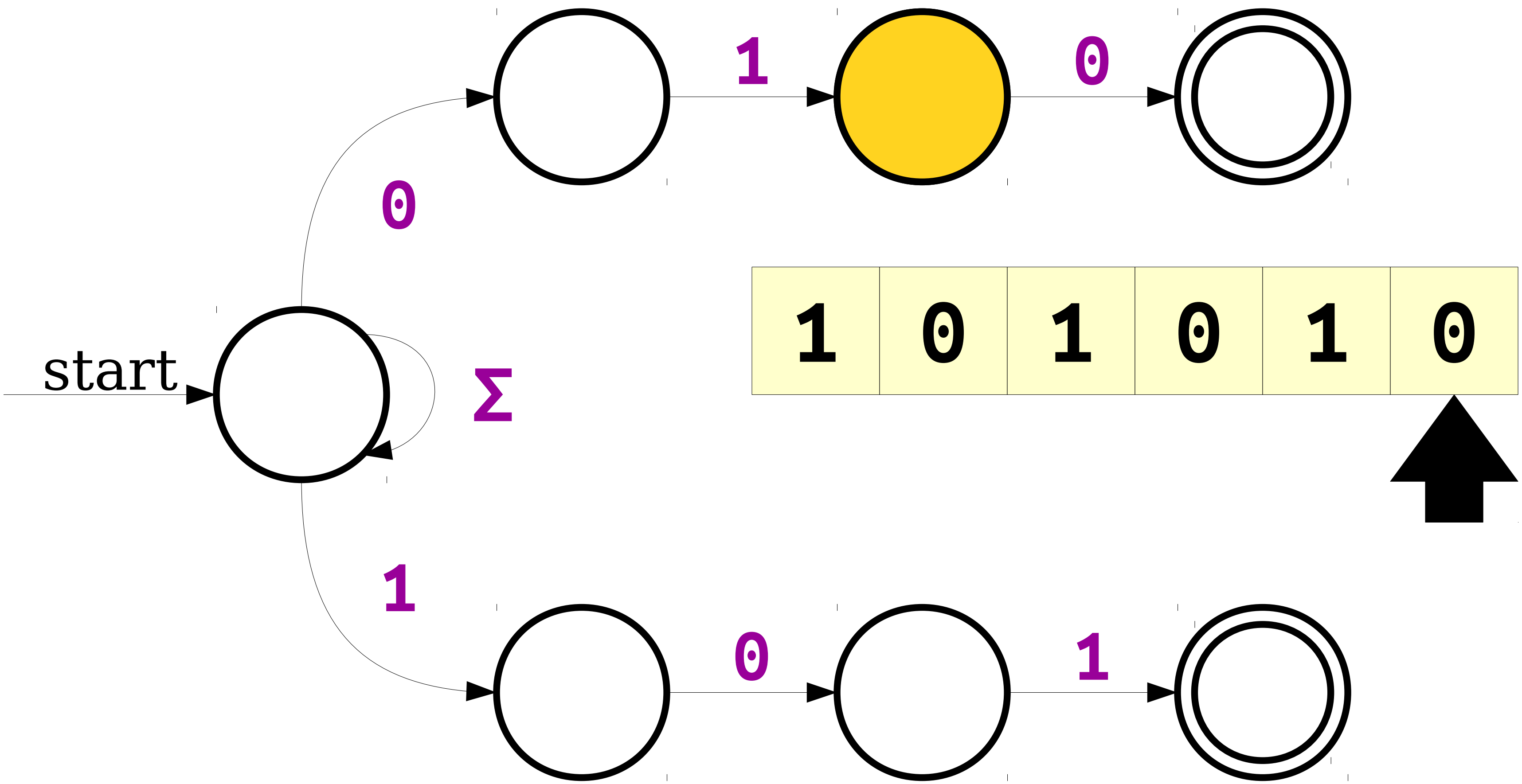
Guess-and-Check

$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$



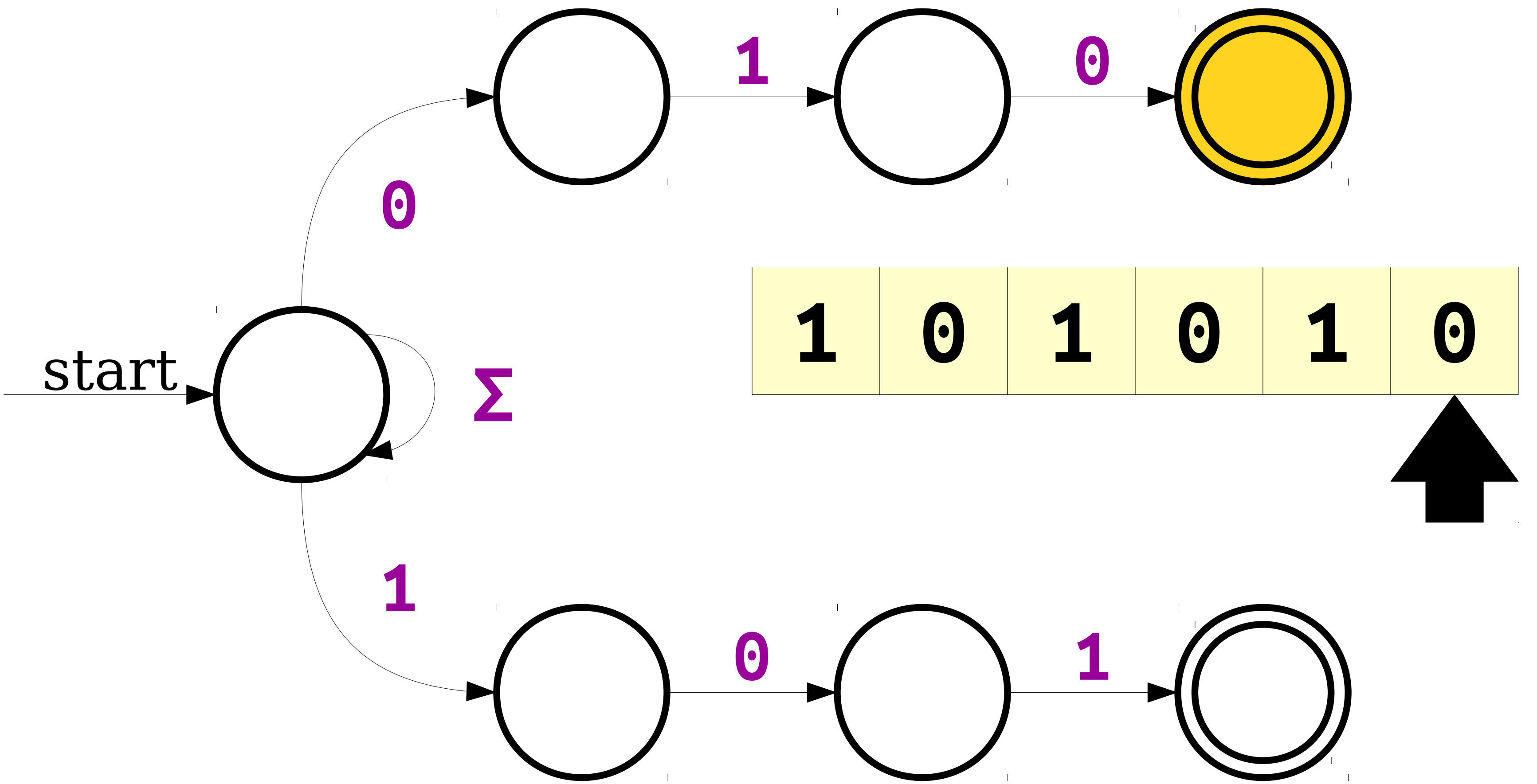
Guess-and-Check

$L = \{ w \in \{0, 1\}^* \mid w \text{ ends in } 010 \text{ or } 101 \}$



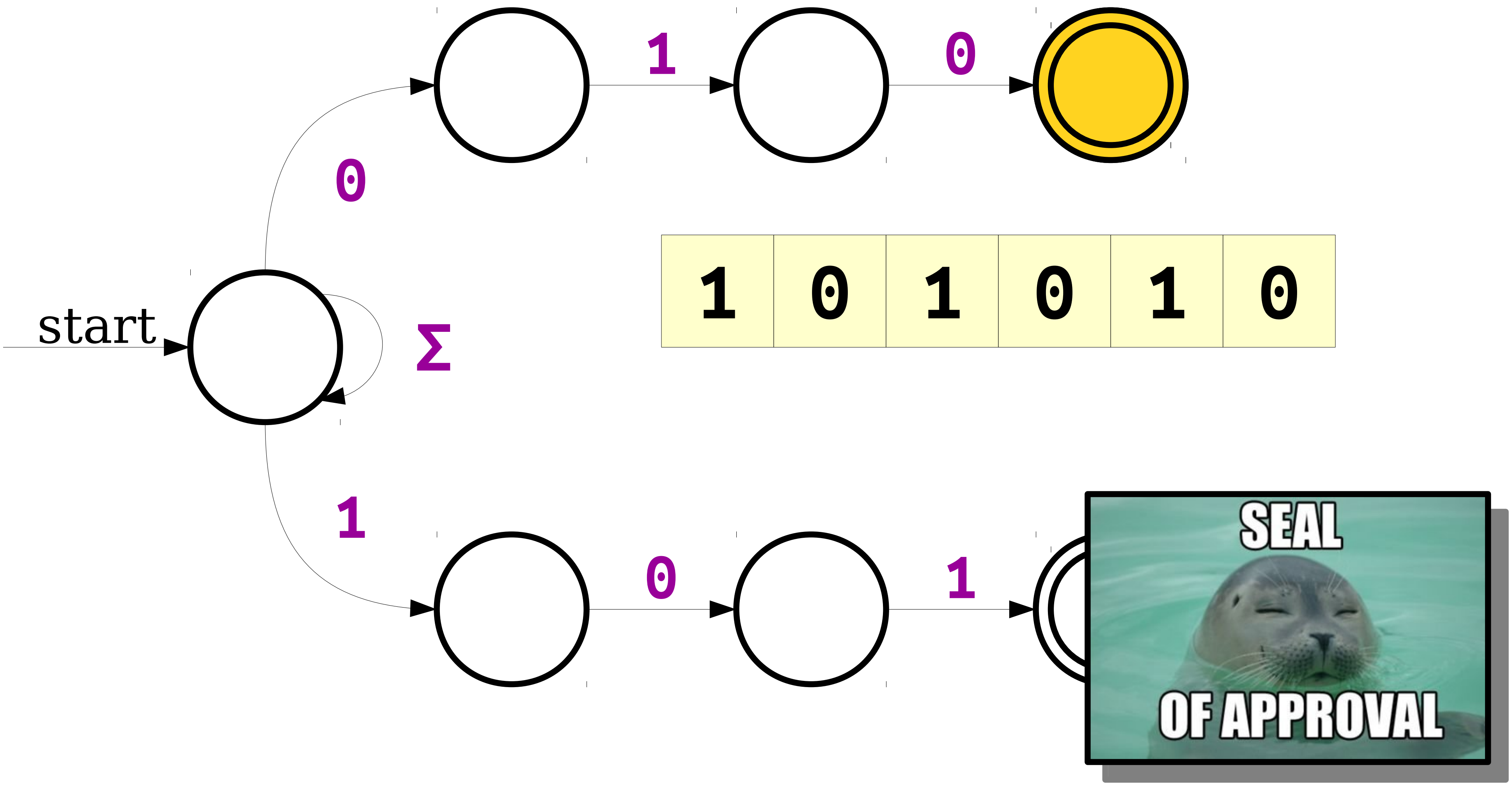
Guess-and-Check

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ ends in } \texttt{010} \text{ or } \texttt{101} \}$



Guess-and-Check

$L = \{ w \in \{\texttt{0}, \texttt{1}\}^* \mid w \text{ ends in } \texttt{010} \text{ or } \texttt{101} \}$

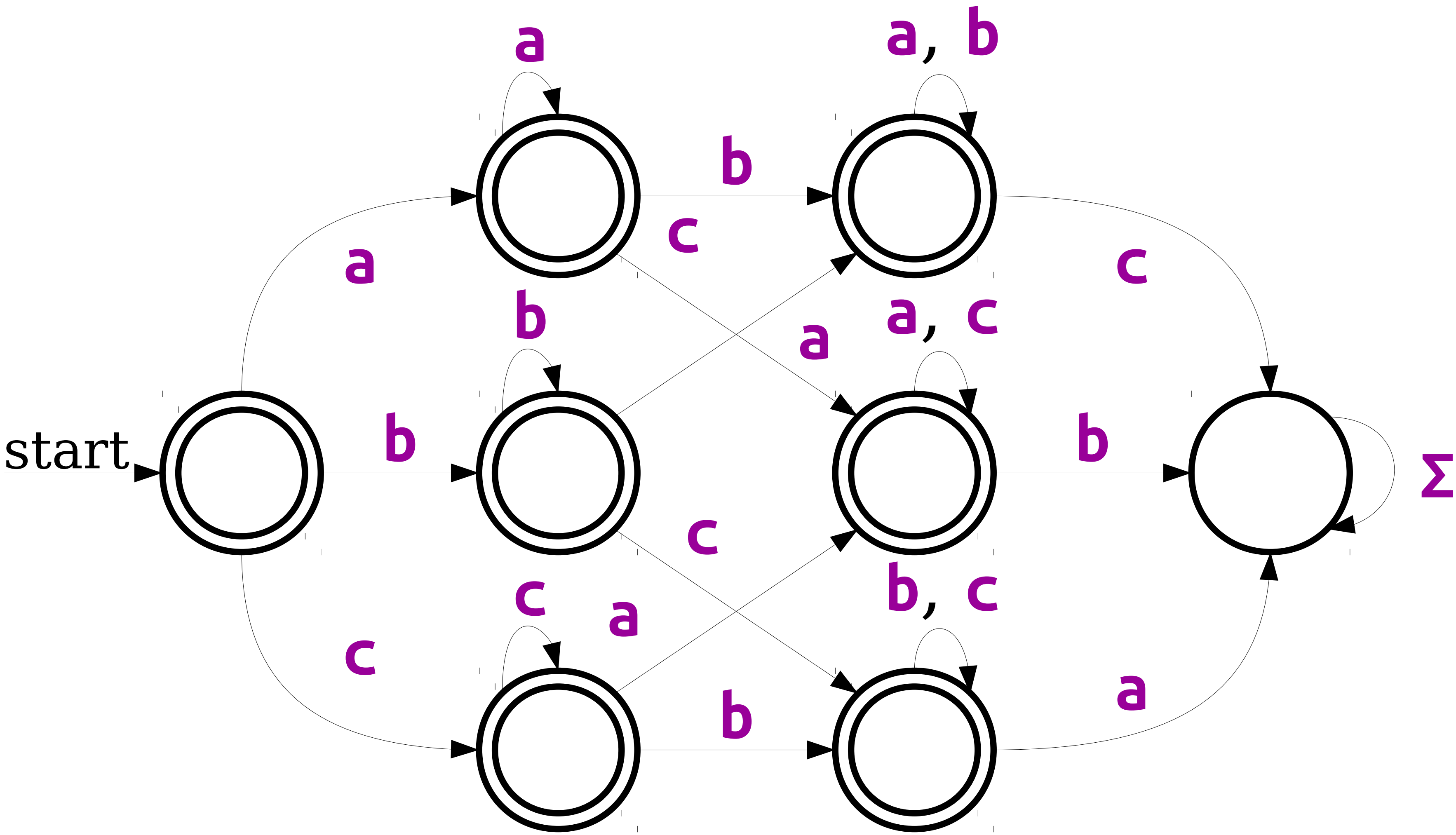


Guess-and-Check

$L = \{\ w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* \mid \text{at least one of } \mathtt{a}, \mathtt{b}, \text{or } \mathtt{c} \text{ is not in } w\ \}$

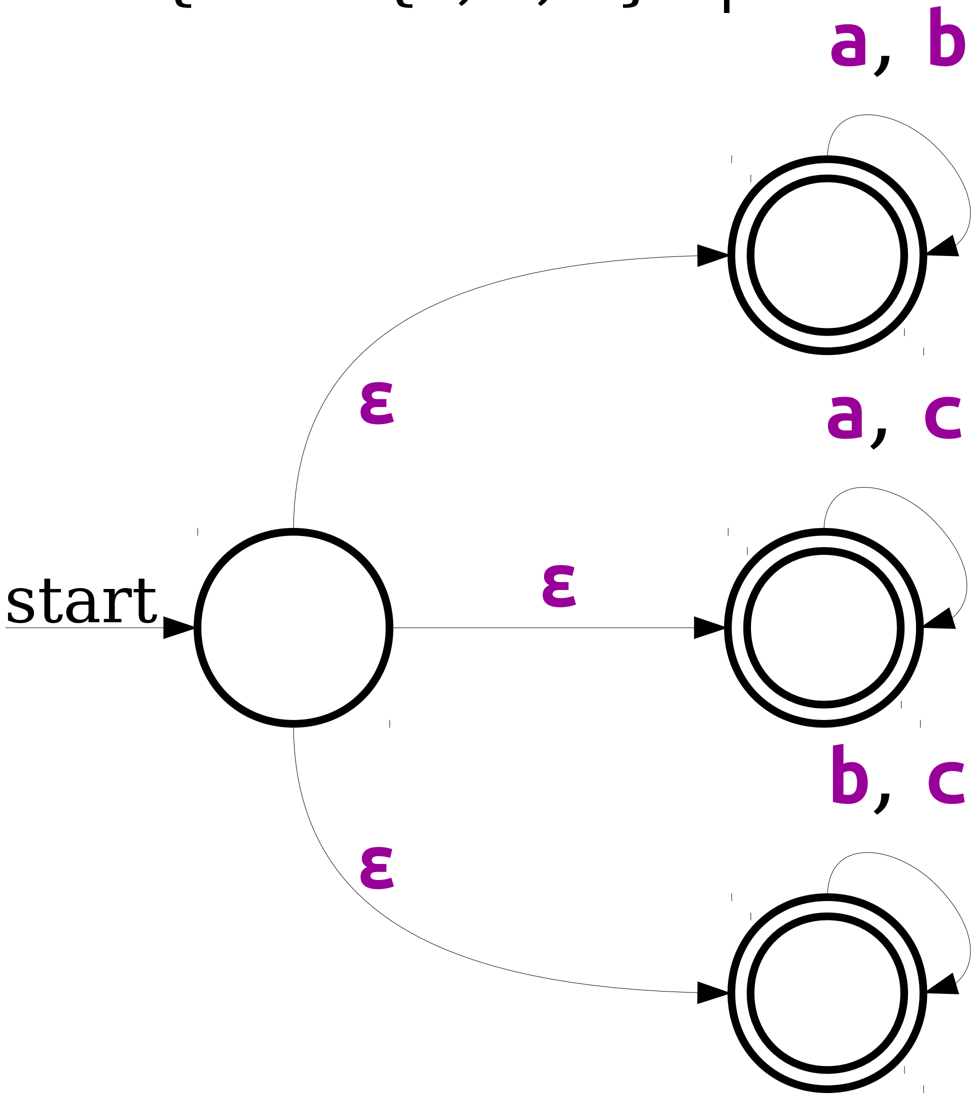
Guess-and-Check

$L = \{\ w \in \{\mathtt{a}, \mathtt{b}, \mathtt{c}\}^* \mid \text{at least one of } \mathtt{a}, \mathtt{b}, \text{or } \mathtt{c} \text{ is not in } w\ \}$



Guess-and-Check

$L = \{ w \in \{$**a**, **b**, **c**$\}^* \mid$ at least one of **a**, **b**, or **c** is not in w $\}$

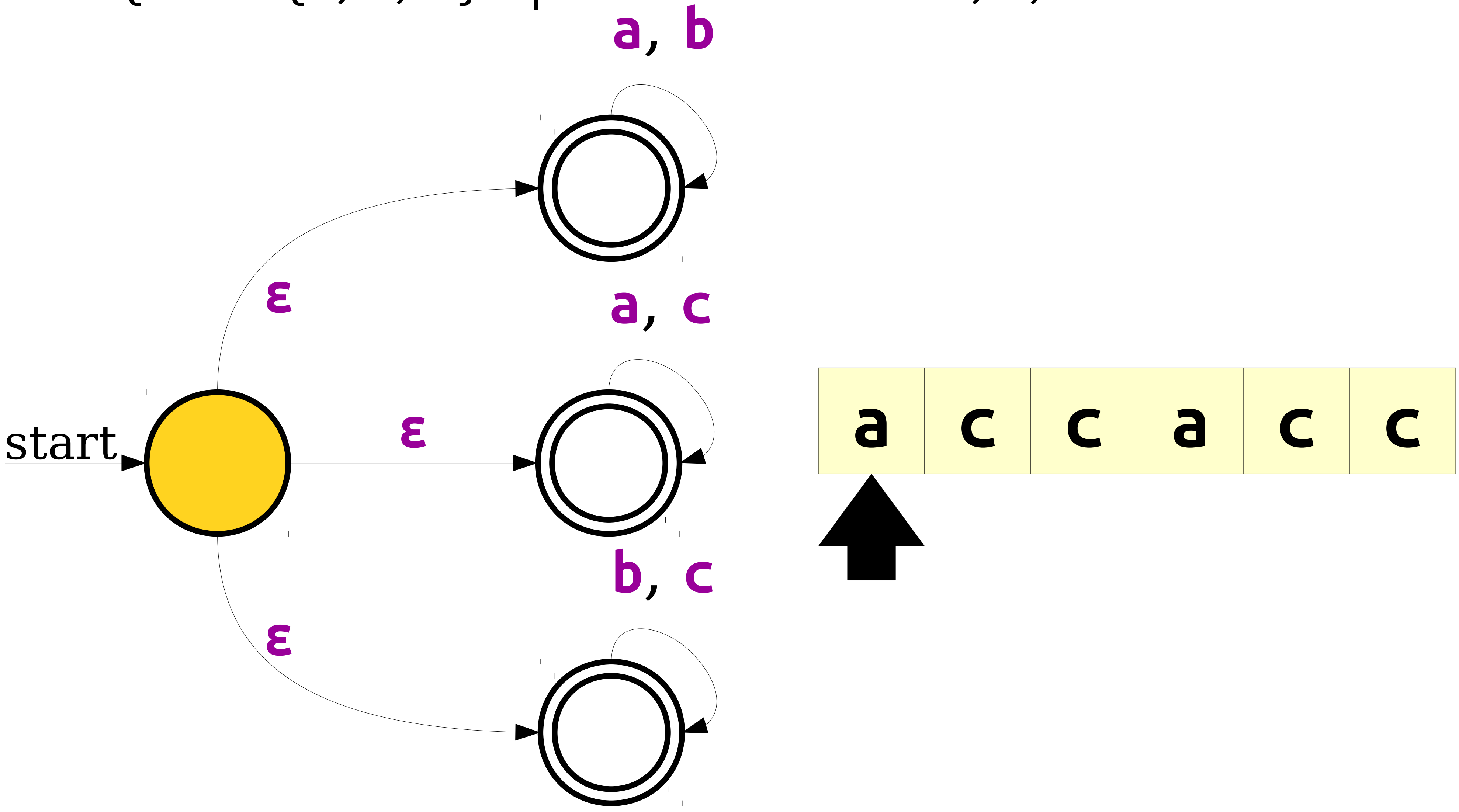


Nondeterministically *guess* which character is missing.

Deterministically *check* whether that character is indeed missing.

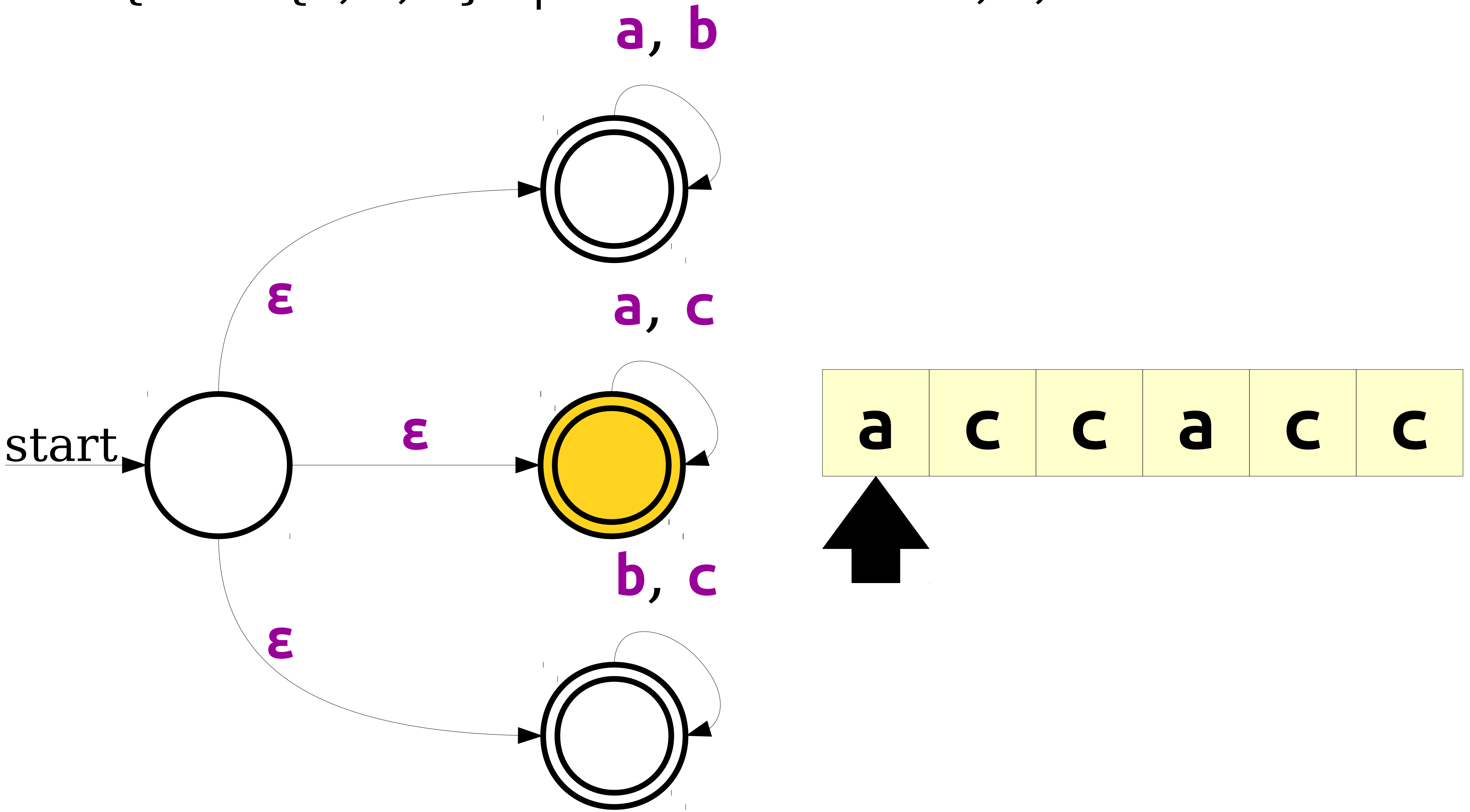
Guess-and-Check

$L = \{ w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* \mid \text{at least one of } \texttt{a}, \texttt{b}, \text{ or } \texttt{c} \text{ is not in } w \}$



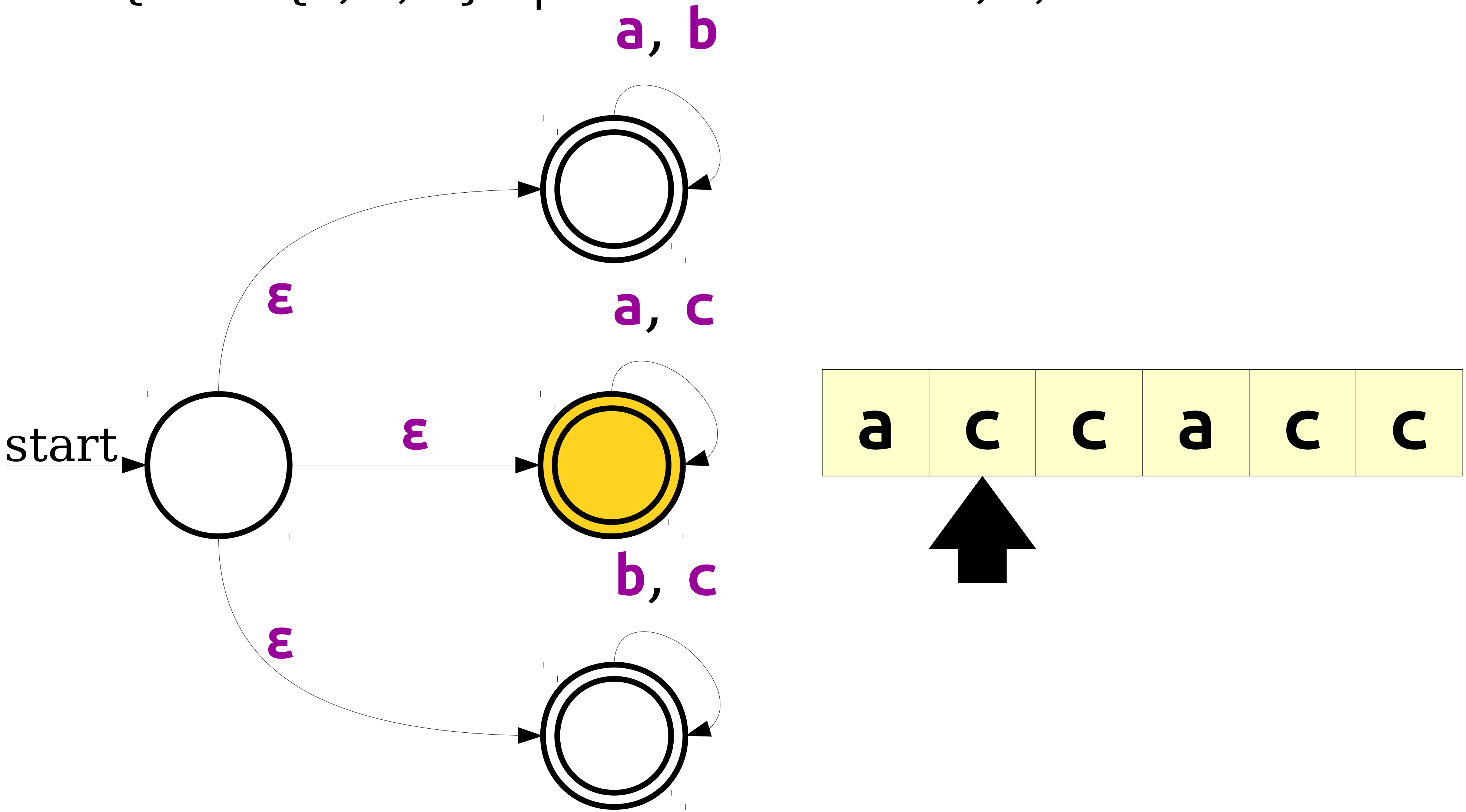
Guess-and-Check

$L = \{ w \in \{$a, b, c$\}^* \mid$ at least one of a, b, or c is not in w $\}$



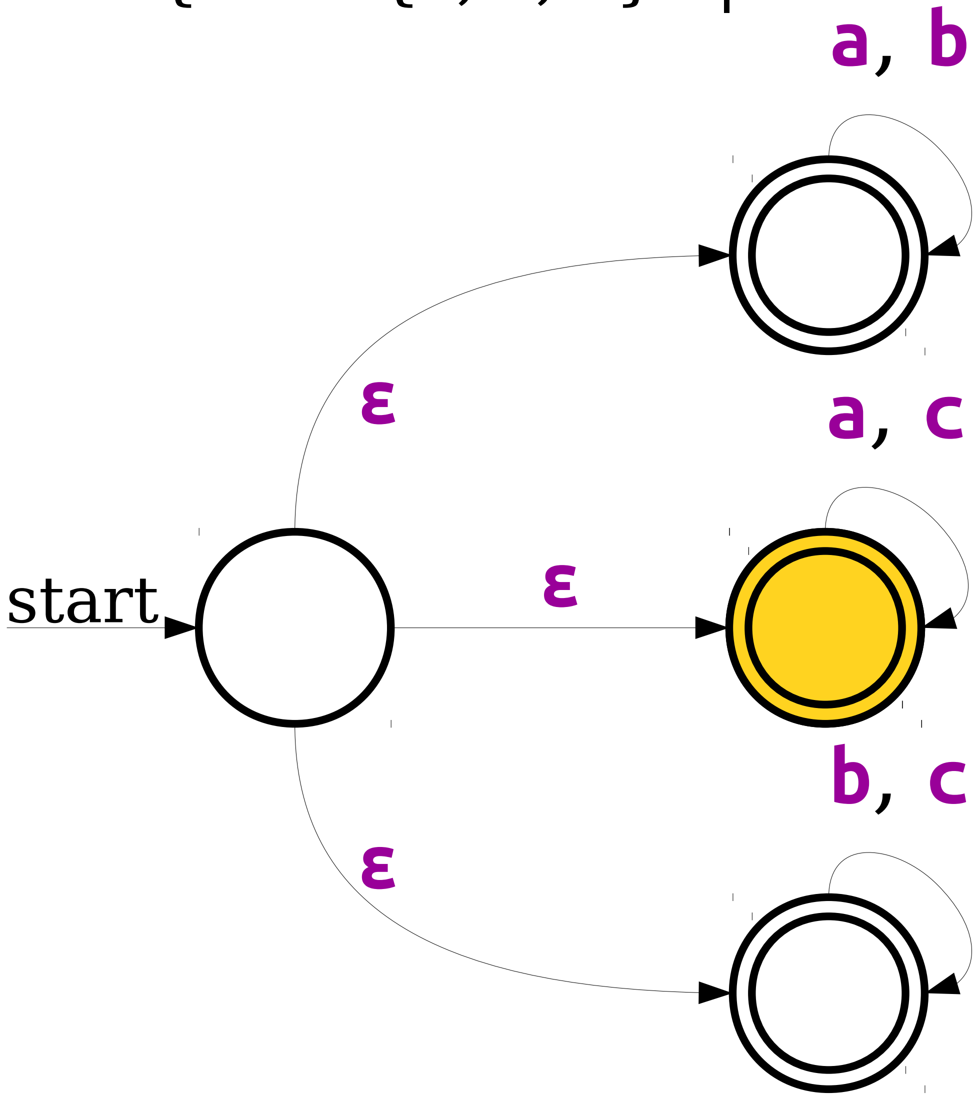
Guess-and-Check

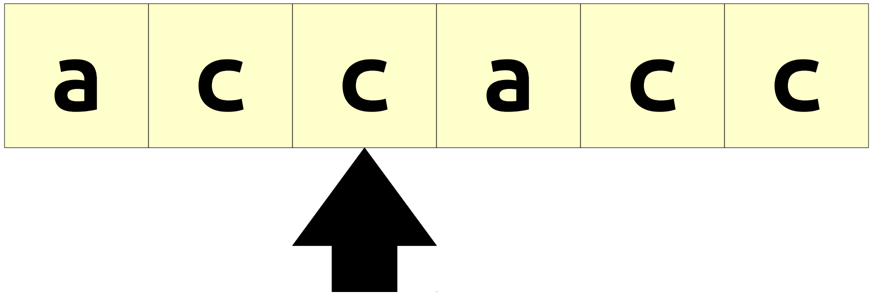
$L = \{ w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* \mid \text{at least one of } \texttt{a}, \texttt{b}, \text{ or } \texttt{c} \text{ is not in } w \}$



Guess-and-Check

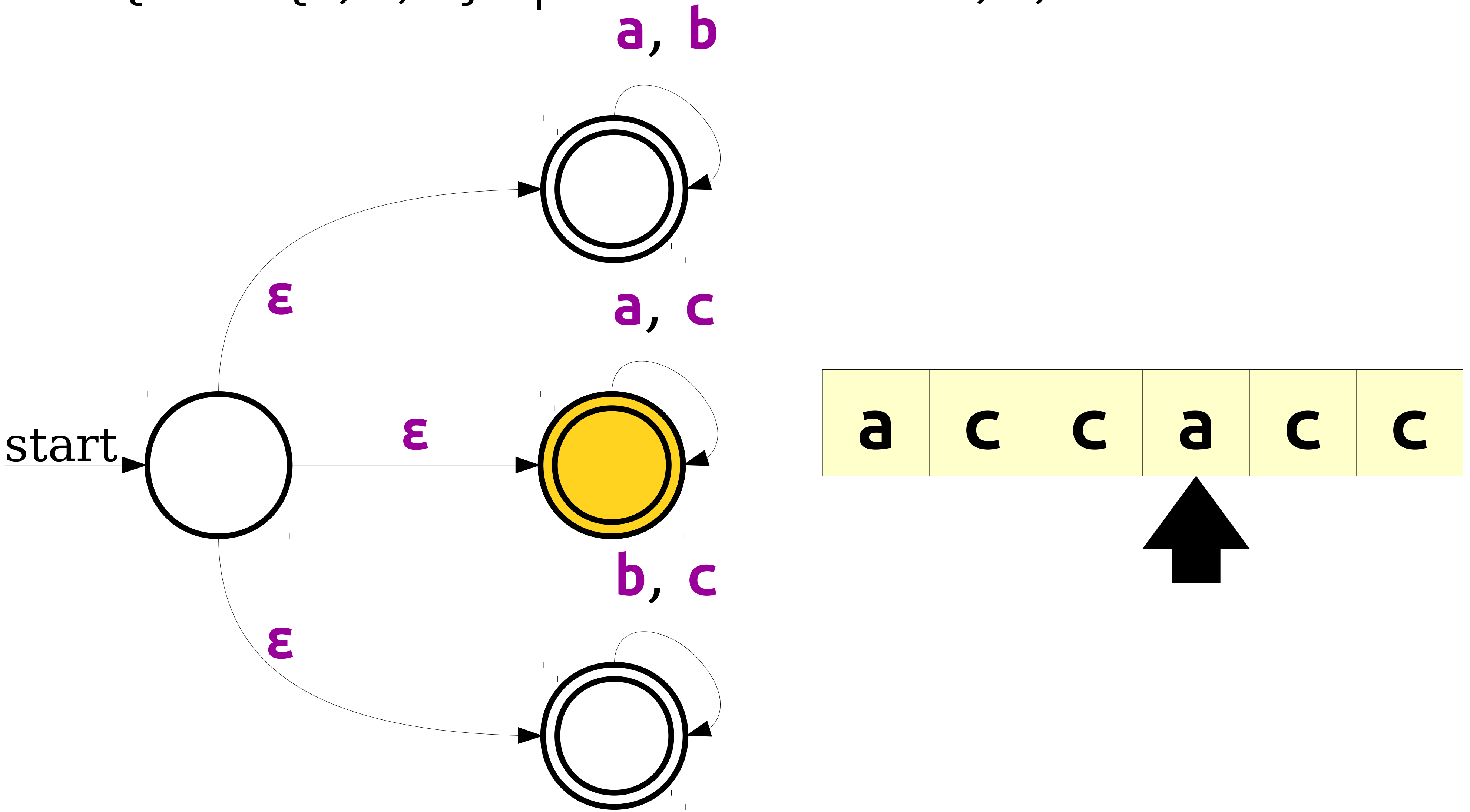
$L = \{ \; w \in \{\texttt{a}, \texttt{b}, \texttt{c}\}^* \mid \text{at least one of } \texttt{a}, \texttt{b}, \text{or } \texttt{c} \text{ is not in } w \; \}$





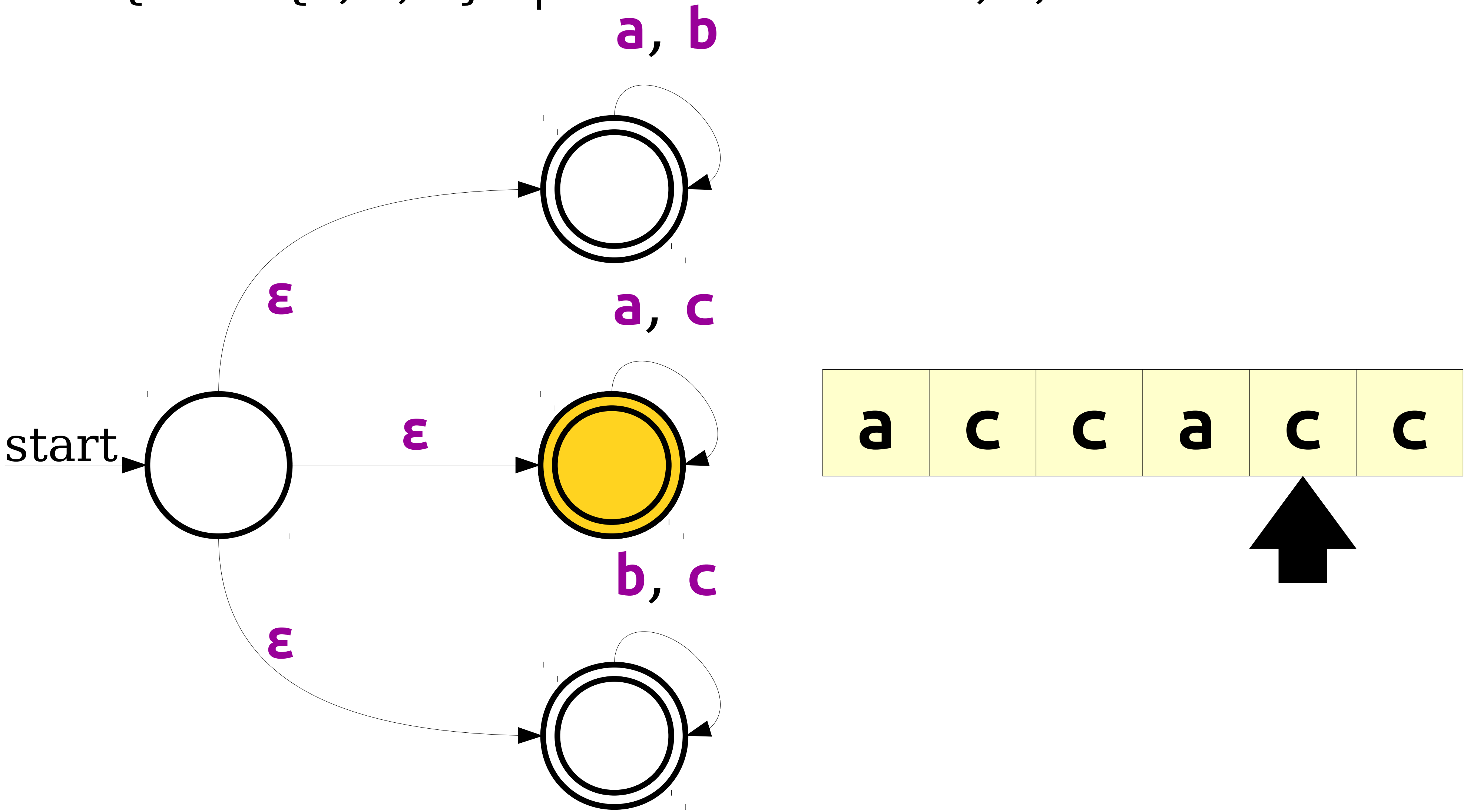
Guess-and-Check

$L = \{ w \in \{$**a**, **b**, **c**$\}^* \mid$ at least one of **a**, **b**, or **c** is not in w $\}$



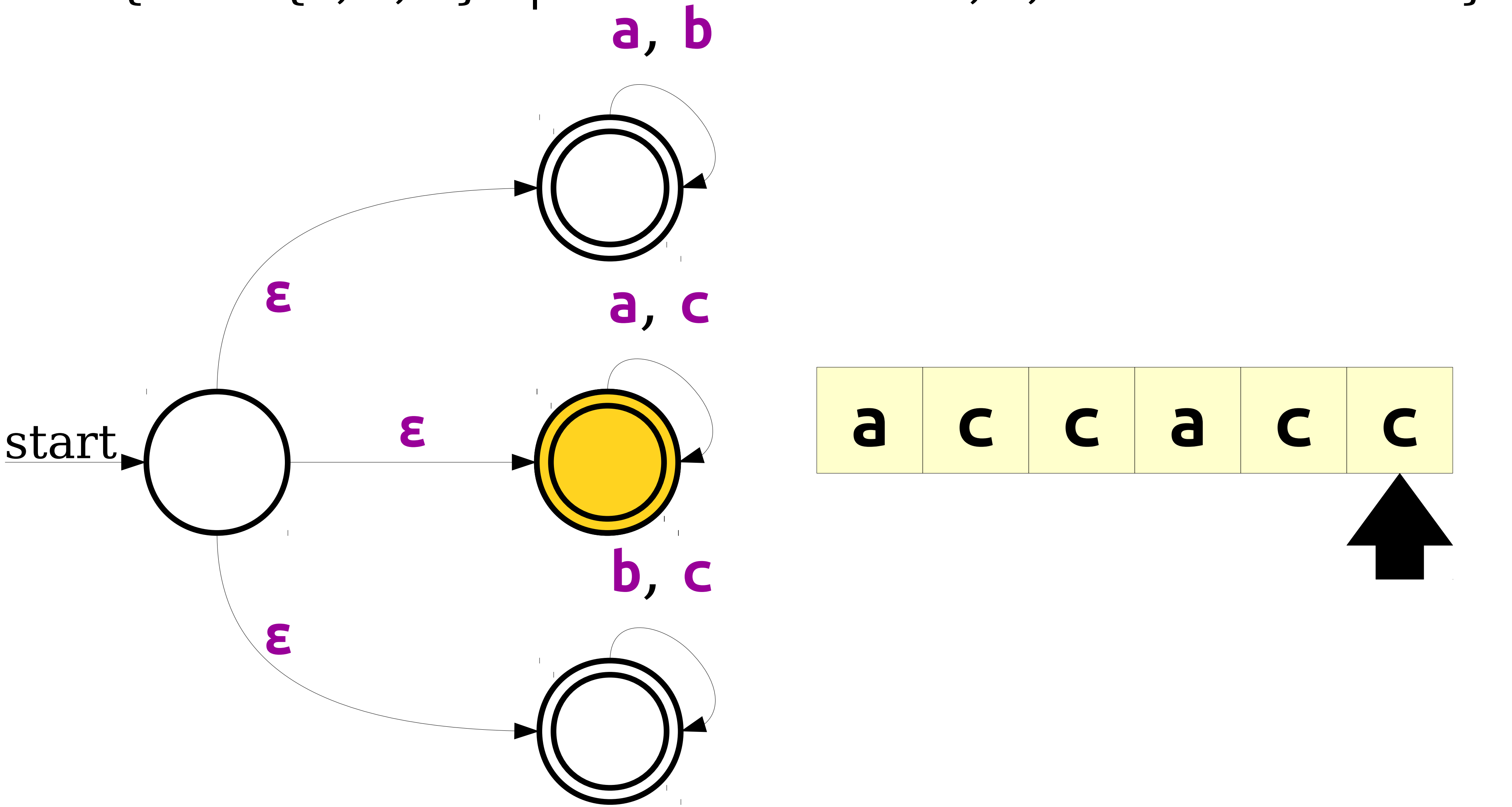
Guess-and-Check

$L = \{ w \in \{$a, b, c$\}^* \mid$ at least one of a, b, or c is not in $w \}$



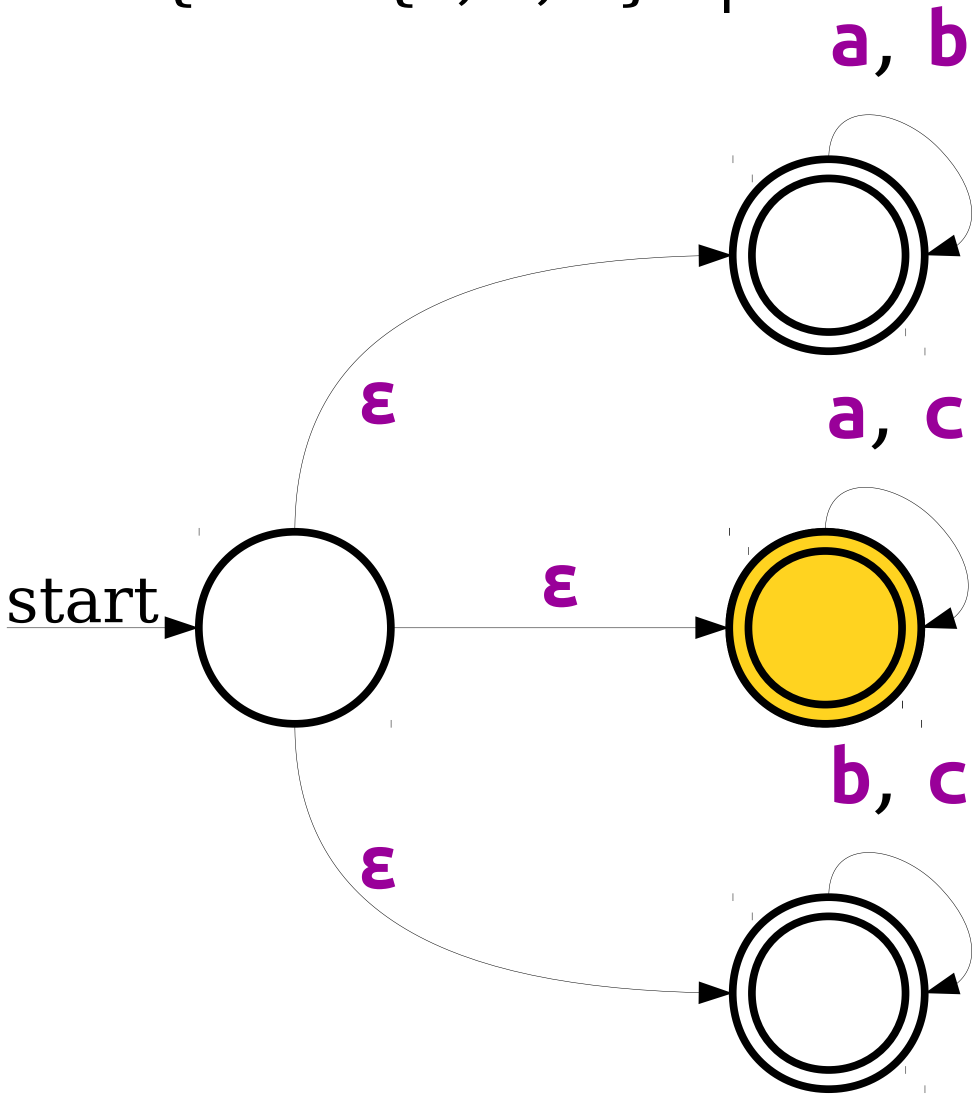
Guess-and-Check

$L = \{ w \in \{$a, b, c$\}^* \mid$ at least one of a, b, or c is not in $w \}$



Guess-and-Check

$L = \{\ w \in \{$**a**, **b**, **c**$\}^* \mid$ at least one of **a**, **b**, or **c** is not in $w\ \}$



a	c	c	a	c	c

Just how powerful are NFAs?

Next Time

- ***The Subset Construction***
  - So beautiful. So elegant. So cool!
- ***Closure Properties of Regular Languages***
  - Transforming languages by transforming machines.
- ***The Kleene Closure***
  - What's the deal with the notation Σ^*?